HOMEWORK SOLUTIONS #5

1. (a) Starting with $y(t) = \int_{-\infty}^{\infty} h(v)x(t-v)dv$, we have

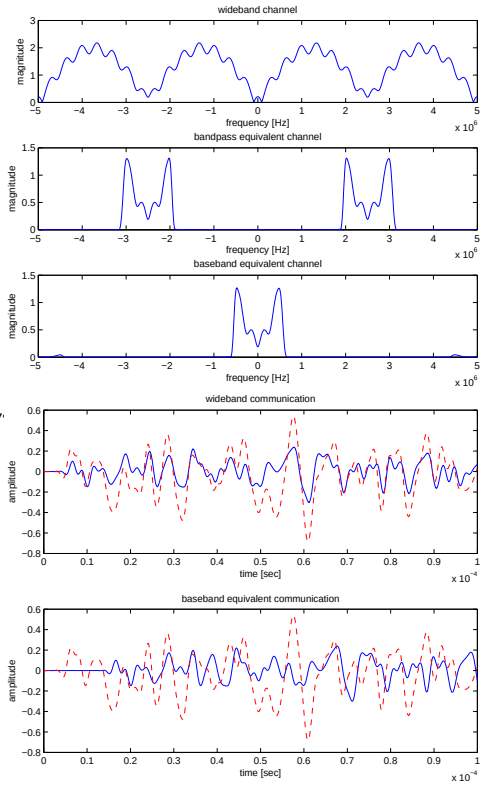
$$\begin{aligned} R_y(\tau) &= \mathrm{E}\{y(t)y^*(t-\tau)\} && (1)\\ &= \mathrm{E}\left\{\int_{-\infty}^{\infty} h(v)x(t-v)dv \int_{-\infty}^{\infty} h^*(u)x^*(t-\tau-u)du\right\} && (2)\\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(v)h^*(u)\,\mathrm{E}\left\{x(t-v)x^*(t-\tau-u)\right\}dvdu && (3)\\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(v)h^*(u)R_x(\tau+u-v)dvdu && (4)\end{aligned}$$

   (b) Since we know that $R_y(\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(v)h^*(u)R_x(\tau+u-v)dvdu$, we can write

$$\begin{aligned} S_y(f) &= \int_{-\infty}^{\infty} R_y(\tau)e^{-j2\pi f\tau}d\tau && (5)\\ &= \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(v)h^*(u)R_x(\tau+u-v)dvdu\right]e^{-j2\pi f\tau}d\tau && (6)\\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(v)e^{-j2\pi fv}h^*(u)e^{j2\pi fu}R_x(\tau+u-v)e^{-j2\pi f(\tau+u-v)}d\tau dvdu && (7)\\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(v)e^{-j2\pi fv}h^*(u)e^{j2\pi fu}\left[\int_{-\infty}^{\infty} R_x(\tau+u-v)e^{-j2\pi f(\tau+u-v)}d\tau\right]dvdu && (8)\\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(v)e^{-j2\pi fv}h^*(u)e^{j2\pi fu}\left[\int_{-\infty}^{\infty} R_x(\tau')e^{-j2\pi f\tau'}d\tau'\right]dvdu && (9)\\ &= S_x(f)\int_{-\infty}^{\infty} h(v)e^{-j2\pi fv}dv\int_{-\infty}^{\infty} h^*(u)e^{j2\pi fu}du && (10)\\ &= S_x(f)\int_{-\infty}^{\infty} h(v)e^{-j2\pi fv}dv\left[\int_{-\infty}^{\infty} h(u)e^{-j2\pi fu}du\right]^* && (11)\\ &= S_x(f)H(f)H^*(f) && (12)\\ &= S_x(f)|H(f)|^2 && (13)\end{aligned}$$

2. The matlab code and plots for this problem appear below:

```
% global parameters
t_max = 100e-6;
Ts = 1/10e6;
t = 0:Ts:t_max;
fc = 2.5e6;
W = 500e3;

% wideband channel
c1 = 1; tau1 = 0;
c2 = -0.99; tau2 = 0.4e-6;
c3 = 0.2; tau3 = 3e-6;
h = zeros(1,round(t_max/Ts)+1);
h(round(tau1/Ts)+1) = c1/Ts;
h(round(tau2/Ts)+1) = c2/Ts;
h(round(tau3/Ts)+1) = c3/Ts;
figure(1)
subplot(311);
plottf(h,Ts,'f');
title('wideband channel')

% bandpass equiv channel
tb = 100*Ts;
Bl = (fc-W)*2*Ts;
Bh = (fc+W)*2*Ts;
b = firls(round(2*tb/Ts),[0,0.95*Bl,Bl,Bh,Bh*1.05,1],[0,0,1,1,0,0]
h_bp = filter(b,1,h)*Ts;
subplot(312);
plottf(h_bp,Ts,'f');
title('bandpass equivalent channel')

% complex baseband channel
tlpf = 10*Ts;
lpf = fir2(round(2*tlpf/Ts),[0,W*2*Ts,(2*fc-W)*2*Ts,1],[1,1,0,0])/
h_bb = filter(lpf,1,h_bp.*exp(-j*2*pi*fc*t))*Ts;
subplot(313);
plottf(h_bb,Ts,'f');
title('baseband equivalent channel')

% generate message signal
randn('state',0);
tl = 50*Ts;
l = fir2(2*tl/Ts,[0,0.25*W*2*Ts,W*2*Ts,1],[1,1,0,0])/Ts;
mt = filter(l,1,randn(1,round(t_max/Ts)+1))*Ts;

% AM communication over wideband channel
s = real(mt.*exp(j*2*pi*fc*t));
x = filter(h,1,s)*Ts;
v = real( filter(lpf,1,2*x.*exp(-j*2*pi*fc*t))*Ts );
figure(2)
subplot(211)
plottf(v,Ts,'t');
hold on; handy = plottf(mt,Ts,'t'); hold off;
set(handy,'LineStyle','--','Color','Red');
title('wideband communication');

% AM communication over complex baseband equivalent channel
vt = filter(h_bb,1,mt)*Ts;
subplot(212)
plottf(real(vt),Ts,'t');
hold on; handy = plottf(mt,Ts,'t'); hold off;
set(handy,'LineStyle','--','Color','Red');
title('baseband equivalent communication');
```



(b) The magnitude response of the bandpass equivalent channel equals that of the wideband channel over the frequency range of transmitted signal, $[-f_c - W, -f_c + W] \cup [f_c - W, f_c + W]$.

(c) The magnitude response of the complex baseband equivalent channel over the range $[-W, W]$ equals that of the wideband and passband equivalent channels over the range $[f_c - W, f_c + W]$.

(e) The recovered signal looks like a highpass filtered version of the trasmitted signal, in that the fast signal variations have been preserved, but not the slow ones.

(f) The complex baseband equivalent system output is identical (except for a delay) to the wideband system output. The delay is due to the group delay of the filtering used to create the complex baseband channel.