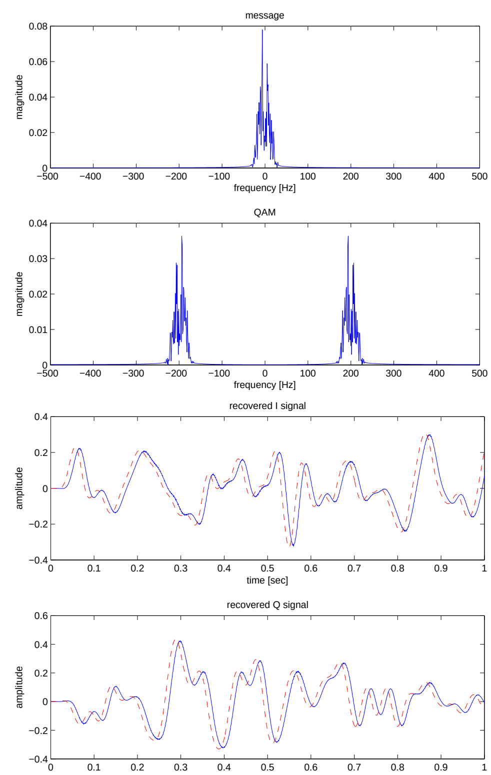
HOMEWORK SOLUTIONS #4

1. From the definition of the Fourier transform and Euler's equation we have

$$
\begin{aligned}
\mathcal{F}\{\cos(2\pi f_c t)c(t)\} &= \int_{-\infty}^{\infty} c(t)\cos(2\pi f_c t)e^{-j2\pi f t}dt = \frac{1}{2}\int_{-\infty}^{\infty} c(t)\left[e^{-j2\pi f_c t} + e^{j2\pi f_c t}\right]e^{-j2\pi f t}dt \quad (1)\\
&= \frac{1}{2}\int_{-\infty}^{\infty} c(t)e^{-j2\pi(f+f_c)t}dt + \frac{1}{2}\int_{-\infty}^{\infty} c(t)e^{-j2\pi(f-f_c)t}dt \quad (2)\\
&= \frac{1}{2}\Big[C(f+f_c) + C(f-f_c)\Big] \quad (3)
\end{aligned}
$$

2. The code and plots for the Complex-Baseband QAM experiment appear below.

```
% generate messages
Ts = 1/1000;
t_max = 1.0;
t = 0:Ts:t_max;
W = 25;
to = 50e-3;
h = fir2(2*to/Ts,[0,0.25*W*2*Ts,W*2*Ts,1],[1,1,0,0])/Ts;
mI = filter(h,1,randn(1,t_max/Ts+1))*Ts;
mQ = filter(h,1,randn(1,t_max/Ts+1))*Ts;
m = mI + j*mQ;

% complex-baseband QAM modulate (without pilot)
fc = 200;
s = real( m.*exp(j*2*pi*fc*t) );

% coherently QAM-demodulate to complex-baseband
Bp = W;
Bs = 2*fc-W;
to = 10e-3;
b = fir2(2*to/Ts,[0,Bp*2*Ts,Bs*2*Ts,1],[1,1,0,0])/Ts;
v = filter(b,1, s.*exp(-j*2*pi*fc*t)*2 )*Ts;
vI = real(v);
vQ = imag(v);

% plot results
figure(1)
subplot(211)
 plottf(m,Ts,'f');
 title('message')
subplot(212)
 plottf(s,Ts,'f');
 title('QAM')
figure(2)
subplot(211);
 plottf(vI,Ts,'t');
 hold on; hh = plottf(mI,Ts,'t'); hold off;
 set(hh,'LineStyle','--','Color','Red');
 title('recovered I signal')
subplot(212);
 plottf(vQ,Ts,'t');
 hold on; hh = plottf(mQ,Ts,'t'); hold off;
 set(hh,'LineStyle','--','Color','Red');
 title('recovered Q signal')
```



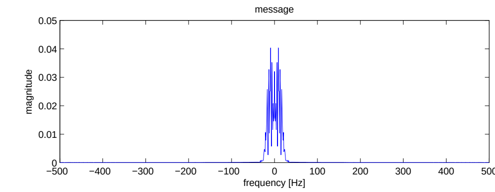
(a) As expected, the message signal bandwidth is approximately $W = 25$ Hz and $|\tilde{M}(f)|$ is not symmetric around $f = 0$ because $\tilde{m}(t)$ is complex-valued.

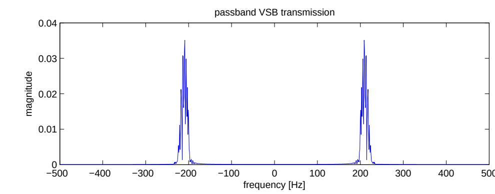
(b) The transmitted signal is a bandpass signal centered at $f_c = 200$ Hz and approximate bandwidth $2W = 50$ Hz, as expected. Also, the basic shape of the message $|\tilde{M}(f)|$ is preserved.

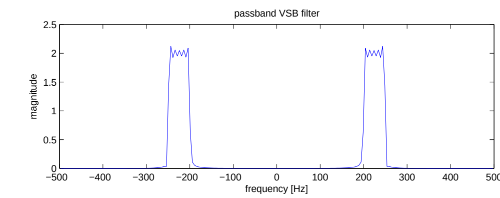
(c) The recovered signals look just like the original messages, but delayed by 10 ms, which was the group delay of the receiver LPF.

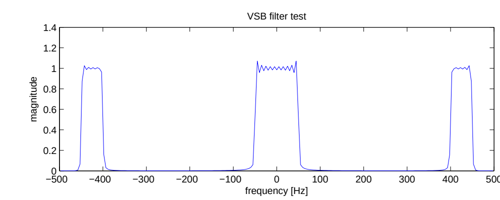
3. The code and plots for the Passband VSB experiment appear below.

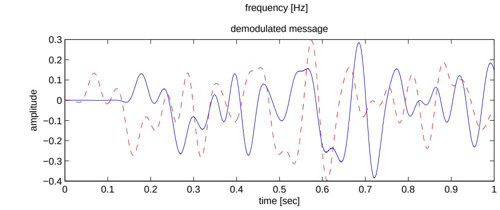
```
% passband VSB mod/demod

randn('state',0);
Ts = 1/1000;

t_max = 1.0;
t = 0:Ts:t_max;
W = 25;
to_h = 50e-3;
Lh = 2*to_h/Ts;
h = fir2(Lh,[0,0.25*W*2*Ts,W*2*Ts,1],[1,1,0,0])/Ts;
m = filter(h,1,randn(1,t_max/Ts+1))*Ts;

fc = 200;
alf = 0.1;
to_c = 100e-3;
Lc = 2*to_c/Ts;
c = firvsb(Lc,alf,1/Ts,W,fc)/Ts;
s = filter(c,1,m.*cos(2*pi*fc*t))*Ts;

Bp = W;
Bs = 2*fc-W;
to_lpf = 10e-3;
Llpf = 2*to_lpf/Ts;
lpf = fir2(Llpf,[0,Bp*2*Ts,Bs*2*Ts,1],[1,1,0,0])/Ts;
v = filter(lpf,1,2*s.*cos(2*pi*fc*(t-to_c)))*Ts;

figure(1);
 subplot(211);
 plottf(m,Ts,'f');
 title('message')
 subplot(212);
 plottf(s,Ts,'f');
 title('passband VSB transmission')
figure(2);
 subplot(211);
 plottf(c,Ts,'f');
 title('passband VSB filter')
 subplot(212);
 plottf(cos(2*pi*fc*Ts*([0:Lc]-Lc/2)).*c,Ts,'f');
 title('VSB filter test')
figure(3);
 plottf(v,Ts);
 hold on; plot(t,m,'r--'); hold off;
 title('demodulated message')
```

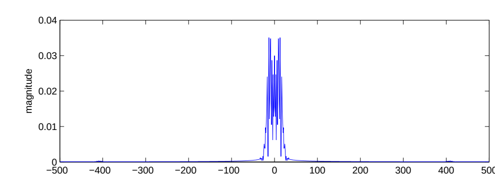












(a) As expected, the message signal bandwidth is approximately $W = 25$ Hz and $|M(f)|$ is symmetric around $f = 0$ because $m(t)$ is complex-valued.

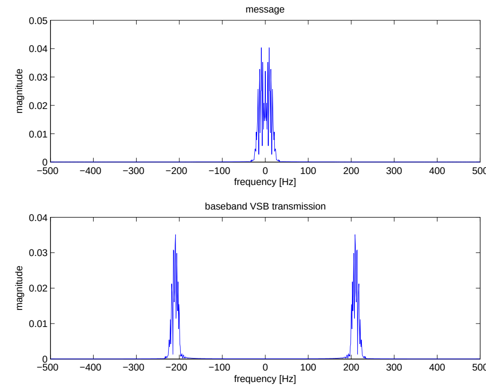
(b) As expected for VSB, the transmitted spectrum over the frequency band $[f_c, f_c + W]$ looks like a shifted version of the message spectrum over the frequency band $[0, W]$, while the transmitted spectrum between DC and just short of f_c is suppressed.

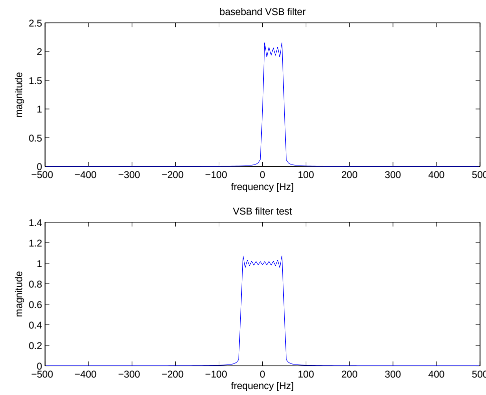
(c) The passband VSB filter frequency response $|C(f)|$ looks good: we can see that it suppresses frequencies between zero and $f_c = 200$ Hz. From the other subplot, we see that $\frac{1}{2}\big|C(f - f_c) + C(f + f_c)\big| \approx 1$ for $|f| \le W$, which confirms that the passband filter is designed properly.

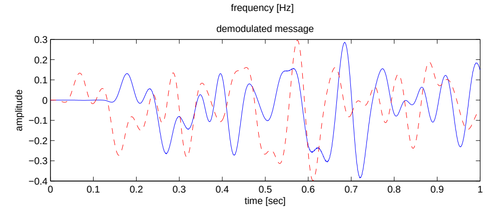
(d) The recovered signal looks just like a delayed version of the original message, indicating that our VSB system is working properly. As expected, the delay looks to be 110 msec, which is the sum of the group delays of the passband VSB filter and the demodulator's LPF.

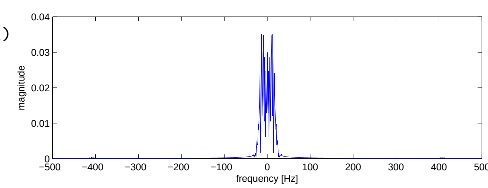
4. The code and plots for the Complex-Baseband VSB experiment appear below.

```
% baseband VSB mod/demod

randn('state',0);
Ts = 1/1000;

t_max = 1.0;
t = 0:Ts:t_max;
W = 25;
to_h = 50e-3;
Lh = 2*to_h/Ts;
h = fir2(Lh,[0,0.25*W*2*Ts,W*2*Ts,1],[1,1,0,0])/Ts;
m = filter(h,1,randn(1,t_max/Ts+1))*Ts;

alf = 0.1;
to_c = 100e-3;
Lc = 2*to_c/Ts;
c = firvsb(Lc,alf,1/Ts,W)/Ts;
fc = 200;
s = real(exp(j*2*pi*fc*t).*filter(c,1,m)*Ts);

Bp = W;
Bs = 2*fc-W;
to_lpf = 10e-3;
Llpf = 2*to_lpf/Ts;
lpf = fir2(Llpf,[0,Bp*2*Ts,Bs*2*Ts,1],[1,1,0,0])/Ts;
v = real( filter(lpf,1,2*s.*exp(-j*2*pi*fc*(t-to_c)))*Ts );

figure(1);
 subplot(211);
 plottf(m,Ts,'f');
 title('message')
 subplot(212);
 plottf(s,Ts,'f');
 title('baseband VSB transmission')
figure(2);
 subplot(211);
 plottf(c,Ts,'f');
 title('baseband VSB filter')
 subplot(212);
 plottf(real(c),Ts,'f');
 title('VSB filter test')
figure(3);
 plottf(v,Ts);
%subplot(212); axe = axis; axis([-2*W,2*W,axe(3:4)]); subplot(211)
 hold on; plot(t,m,'r--'); hold off;
 title('demodulated message')
```









(a) As expected, the message signal bandwidth is approximately $W = 25$ Hz and $|M(f)|$ is symmetric around $f = 0$ because $m(t)$ is complex-valued.

(b) As expected for VSB, the transmitted spectrum over the frequency band $[f_c, f_c + W]$ looks like a shifted version of the message spectrum over the frequency band $[0, W]$, while the transmitted spectrum between DC and just short of f_c is suppressed.

(c) The baseband VSB filter frequency response $|\tilde{C}(f)|$ looks good: it suppresses content where $f < 0$ and passes content where $f \geq 0$. (Only a complex-valued impulse response could do this!) From the other subplot, we see that $\frac{1}{2}\big|\tilde{C}(f)+\tilde{C}^*(-f)\big| \approx 1$ for $|f| \leq W$, which confirms that the baseband filter is designed properly.

(d) The recovered signal looks just like a delayed version of the original message, indicating that our VSB system is working properly. As expected, the delay looks to be 110 msec, which is the sum of the group delays of the baseband VSB filter and the demodulator's LPF.