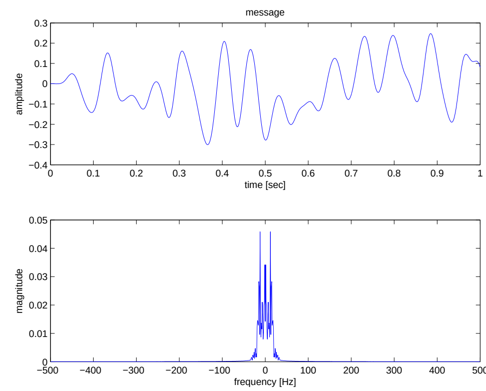
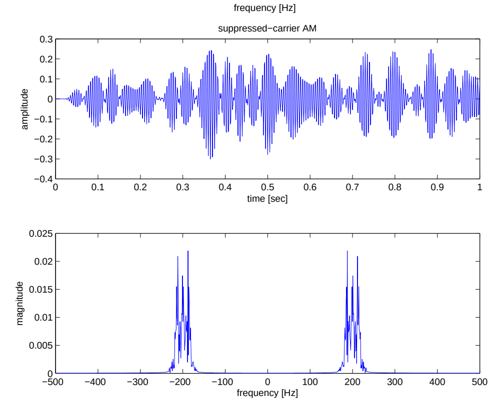
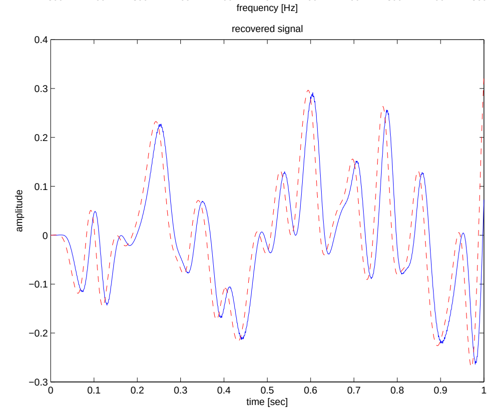
HOMEWORK SOLUTIONS #3

1. The code and plots for the coherent AM experiment appear below.







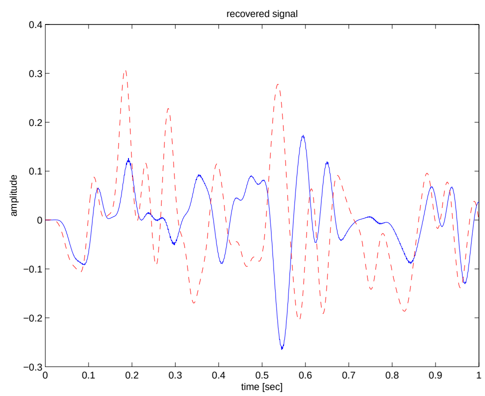

```
% generate message
Ts = 1/1000;
t_max = 1.0;
t = 0:Ts:t_max;
W = 25;
to = 50e-3;
h = fir2(2*to/Ts,[0,0.25*W*2*Ts,W*2*Ts,1],[1,1,0,0])/Ts;
m = filter(h,1,randn(1,t_max/Ts+1))*Ts;

% AM modulate without pilot
fc = 200;
s = m.*cos(2*pi*fc*t);

% AM demodulate coherently
Bp = W;
Bs = 2*fc-W;
to = 10e-3;
b = fir2(2*to/Ts,[0,Bp*2*Ts,Bs*2*Ts,1],[1,1,0,0])/Ts;
fo = 0;
v = filter(b,1, s.*cos(2*pi*(fc+fo)*t)*2 )*Ts;

% plot results
figure(1)
 plottf(m,Ts);
 title('message')
figure(2)
 plottf(s,Ts);
 title('suppressed-carrier AM')
figure(3)
 plottf(v,Ts,'t');
 hold on; hh = plottf(m,Ts,'t'); hold off;
 set(hh,'LineStyle','--','Color','Red');
 title('recovered signal')
```

(a) The message signal bandwidth is approximately $W = 25$ Hz, as expected.

(b) The transmitted signal is a bandpass signal centered at $f_c = 200$ Hz and approximate bandwidth $2W = 50$ Hz, as expected.

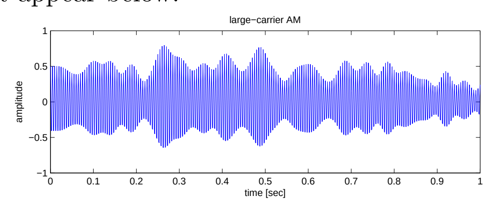
(c) The recovered signal looks just like the original message, but delayed by 10 ms (i.e., the group delay of the receiver LPF).

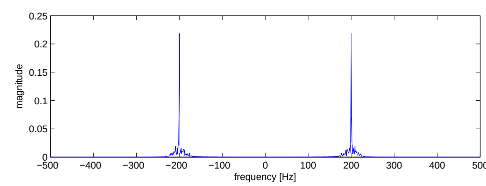
(d) With a carrier frequency offset of $f_o = 1$ Hz, the recovered signal no longer looks like the message $m(t)$, but rather like $m(t)\cos(2\pi f_o t)$:

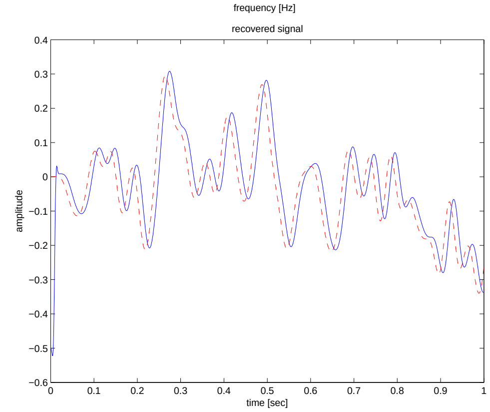


2. The code and plots for the noncoherent AM experiment appear below.

```
% generate message
Ts = 1/1000;
t_max = 1.0;
t = 0:Ts:t_max;
W = 25;
to = 50e-3;
h = fir2(2*to/Ts,[0,0.25*W*2*Ts,W*2*Ts,1],[1,1,0,0])/Ts;
m = filter(h,1,randn(1,t_max/Ts+1))*Ts;

% AM modulate with pilot
fc = 200;
A = 0.5;
s = (m+A).*cos(2*pi*fc*t);

% AM demodulate via envelope detector
Bp = W;
Bs = fc;
to = 10e-3;
b = firls(2*to/Ts,[0,Bp*2*Ts,Bs*2*Ts,1],[1,1,0,0])/Ts;
v = pi/2*Ts*filter(b,1,abs(s));

% plot results
figure(1)
 plottf(s,Ts);
 title('large-carrier AM')
figure(2)
 plottf(v-A,Ts,'t');
 hold on; hh = plottf(m,Ts,'t'); hold off;
 set(hh,'LineStyle','--','Color','Red');
 title('recovered signal')
```



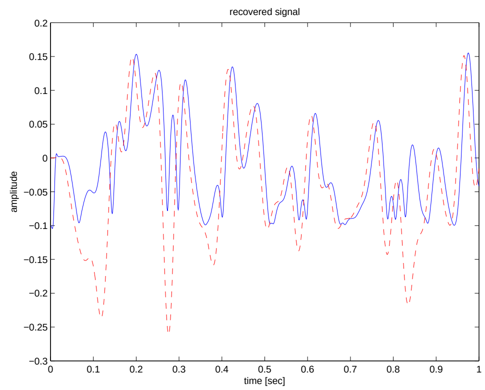




(a) In the time domain, we see that the transmitted signal envelope never gets to zero, which is a requirement for large-carrier AM. In the frequency domain, we see a passband signal centered at $f_c = 200$ Hz with bandwidth $2W$ Hz and a large spike in the middle (representing the pilot tone).

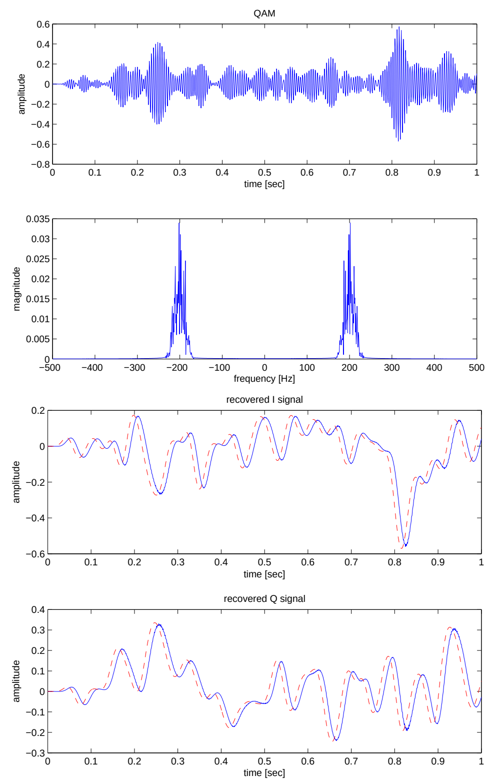
(b) The recovered signal looks just like the original message, but delayed by 10 ms (i.e., the group delay of the receiver LPF).

(c) With a carrier amplitude of $A = 0.1$, we no longer have "large-carrier" AM, and so the envelope detector fails; only the positive half of the message is recovered:



3. The code and plots for the coherent QAM experiment appear below.

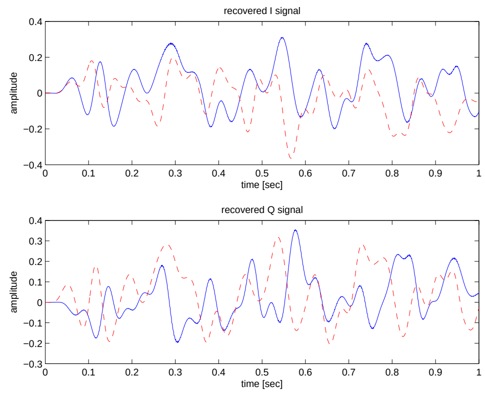
```
% generate message
Ts = 1/1000;
t_max = 1.0;
t = 0:Ts:t_max;
W = 25;
to = 50e-3;
h = fir2(2*to/Ts,[0,0.25*W*2*Ts,W*2*Ts,1],[1,1,0,0])/Ts;
mI = filter(h,1,randn(1,t_max/Ts+1))*Ts;
mQ = filter(h,1,randn(1,t_max/Ts+1))*Ts;

% QAM modulate without pilot
fc = 200;
s = mI.*cos(2*pi*fc*t) - mQ.*sin(2*pi*fc*t);

% QAM demodulate coherently
Bp = W;
Bs = 2*fc-W;
to = 10e-3;
b = fir2(2*to/Ts,[0,Bp*2*Ts,Bs*2*Ts,1],[1,1,0,0])/Ts;
po = pi/2;
vI = filter(b,1, s.*cos(2*pi*fc*t+po)*2 )*Ts;
vQ = filter(b,1, -s.*sin(2*pi*fc*t+po)*2 )*Ts;

% plot results
figure(1)
 plottf(s,Ts);
 title('QAM')
figure(2)
subplot(211);
 plottf(vI,Ts,'t');
 hold on; hh = plottf(mI,Ts,'t'); hold off;
 set(hh,'LineStyle','--','Color','Red');
 title('recovered I signal')
subplot(212);
 plottf(vQ,Ts,'t');
 hold on; hh = plottf(mQ,Ts,'t'); hold off;
 set(hh,'LineStyle','--','Color','Red');
 title('recovered Q signal')
```



(a) The transmitted signal is a bandpass signal centered at $f_c = 200$ Hz and approximate bandwidth $2W = 50$ Hz. As expected, the passband spectrum is not symmetric around its center (as is the case with AM).

(b) The recovered signals looks just like the original messages, but delayed by 10 ms (i.e., the group delay of the receiver LPFs).

(c) With a carrier phase offset of $\pi/2$ radians, the recovered signals no longer look like like the corresponding messages. Instead, it can be seen that $v_{\mathrm{I}}(t) = m_{\mathrm{Q}}(t)$ and $v_{\mathrm{Q}}(t) = -m_{\mathrm{I}}(t)$:



4. (a) A coherent receiver would multiply the received signal by $2\sin(2\pi f_c t)$ and lowpass filter the result using a LPF with passband cutoff $B_p \geq W$ and stopband cutoff $B_s \leq 2f_c - W$:

$$v(t) = \mathrm{LPF}\big\{r(t)\,2\sin(2\pi f_c t)\big\}$$

We can see this via

$$\begin{aligned} v(t) &= \mathrm{LPF}\big\{r(t)\,2\sin(2\pi f_c t)\big\} && (1)\\ &= \mathrm{LPF}\big\{s(t)\,2\sin(2\pi f_c t)\big\} \quad \text{(trivial channel)} && (2)\\ &= \mathrm{LPF}\big\{m(t)\,2\sin^2(2\pi f_c t)\big\} && (3)\\ &= \mathrm{LPF}\big\{m(t) - m(t)\cos(2\pi 2 f_c t)\big\} && (4)\\ &= m(t). && (5) \end{aligned}$$

Since perfect demodulation is possible with $A = 0$, this is the recommended choice since no power is wasted on an unnecessary pilot.

(b) Assuming that $A > \max\{|m(t)|\}$, the only difference between the proposed transmission scheme and large-carrier AM is a transmitted oscillator phase shift of $\pi/2$ radians. Since the envelope detector is blind to phase and frequency offsets, it works both for large-carrier AM and also for the proposed scheme. Thus, the signal can be noncoherently demodulated using

$$v(t) = \frac{\pi}{2}\mathrm{LPF}\big\{|r(t)|\big\} - A$$

as long as $A > \max\{|m(t)|\}$. So that transmission power is not wasted, it is recommended that $A = \max\{|m(t)|\}$.