HOMEWORK SOLUTIONS #2

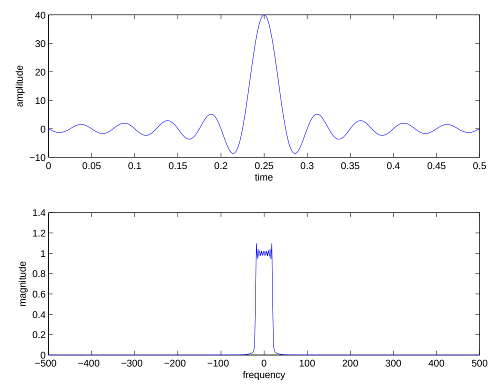
1. (a) Ideal zero-phase LPF:

$$\begin{aligned} h(t) &= \int_{-\infty}^{\infty} H(f) e^{j2\pi ft} df = \int_{-B}^{B} e^{j2\pi ft} df \quad \text{via the definition of } H(f) & (1)\\ &= \frac{1}{j2\pi t} e^{j2\pi ft}\Big|_{f=-B}^{B} = \frac{1}{\pi t}\frac{1}{j2}\Big[e^{j2\pi Bt} - e^{-j2\pi Bt}\Big] = \frac{1}{\pi t}\sin(2\pi Bt) & (2)\\ &= 2B\frac{\sin(2\pi Bt)}{2\pi Bt} = 2B\,\text{sinc}(2Bt). & (3) \end{aligned}$$

   (b) Say $H(f)$ is the ideal zero-phase LPF, as above. Then we can write the ideal linear-phase LPF as $G(f) = H(f)e^{-j2\pi f t_o}$. Since we know that $h(t) = 2B\,\text{sinc}(2Bt)$, we can use the fact that $X(f)e^{-j2\pi f t_o} \overset{\mathcal{F}}{\longleftrightarrow} x(t - t_0)$ to claim that $g(t) = 2B\,\text{sinc}(2B(t - t_o))$.

2. (a) Truncated-sinc LPF:

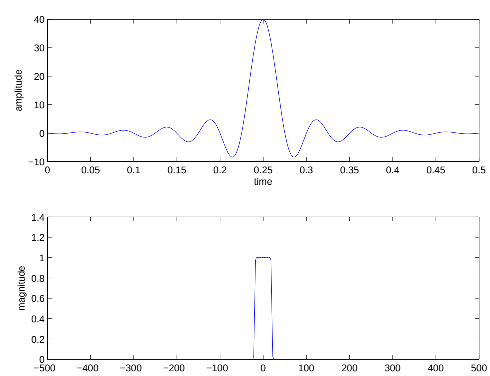
```
Ts = 0.001;
t_o = 0.25;
t = 0:Ts:2*t_o;
B = 20;
h = 2*B*sinc(2*B*(t-t_o));
plottf(h,Ts)
```



The magnitude response is not perfect in that there is ringing near the cutoff frequency.

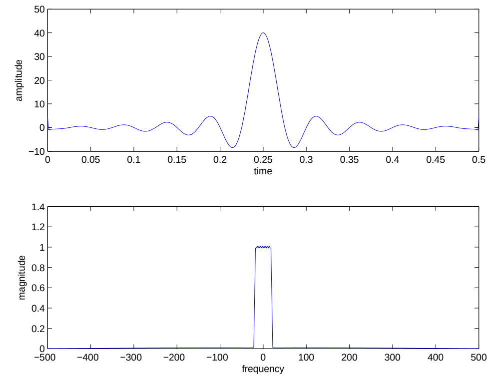
   (b) `firls`-designed LPF:

```
Ts = 0.001;
t_o = 0.25;
Lf = 2*t_o/Ts;
B = 20;
fp = 0.9*B*2*Ts;
fs = 1.1*B*2*Ts;
h = firls(Lf,[0,fp,fs,1],[1,1,0,0])/Ts;
plottf(h,Ts);
```



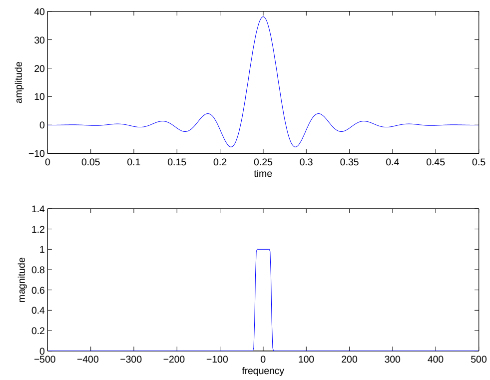
(c) `firpm`-designed LPF:

```
Ts = 0.001;
t_o = 0.25;
Lf = 2*t_o/Ts;
B = 20;
fp = 0.9*B*2*Ts;
fs = 1.1*B*2*Ts;
h = firpm(Lf,[0,fp,fs,1],[1,1,0,0])/Ts;
plottf(h,Ts);
```



(d) `fir2`-designed LPF:

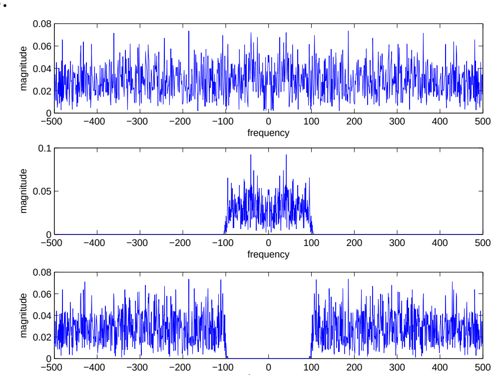
```
Ts = 0.001;
t_o = 0.25;
Lf = 2*t_o/Ts;
B = 20;
fp = 0.9*B*2*Ts;
fs = 1.1*B*2*Ts;
h = fir2(Lf,[0,fp,fs,1],[1,1,0,0])/Ts;
plottf(h,Ts);
```



(e) The LPFs designed using the MATLAB built-in routines yield magnitude responses that are generally much closer to ideal than the truncated-sinc LPF. The passbands from `firls` and `fir2` are very flat, that from `firpm` has very small ripples, while that from the truncated-sinc filter is flat except for severe ringing near the passband edge. The stopbands from `firls` and `fir2` are essentially zero over the desired range, that of the truncated-sinc filter is also zero except near the stopband edge, while that from `firpm` doesn't quite reach zero, which could be problematic. The impulse responses of all the filters look pretty similar, except that the MATLAB built-in LPFs have impulse responses that decay smoothly to zero in comparison to the truncated-sinc LPF.

3. The original noise waveform and its LPF'ed and HPF'ed versions are shown below, along with the MATLAB code that designs and implements the filters:

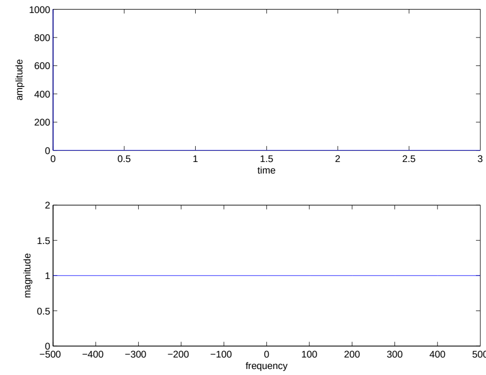
```
Ts = 0.001;
t_o = 0.25;
Lf = 2*t_o/Ts;
B = 100;
fp = 0.9*B*2*Ts;
fs = 1.1*B*2*Ts;
h_lpf = firls(Lf,[0,fp,fs,1],[1,1,0,0])/Ts;
h_hpf = firls(Lf,[0,fp,fs,1],[0,0,1,1])/Ts;
t_max = 1;
x = randn(1,t_max/Ts);
y_lpf = Ts*conv(h_lpf,x);
y_hpf = Ts*conv(h_hpf,x);
subplot(3,1,1); plottf(x,Ts,'f');
subplot(3,1,2); plottf(y_lpf,Ts,'f');
subplot(3,1,3); plottf(y_hpf,Ts,'f');
```



The filtering works as expected: it preserves the input signal over its passband but rejects it over its stopband.

4. (a) The approximate Dirac delta and its `plottf`-approximated Fourier transform are:

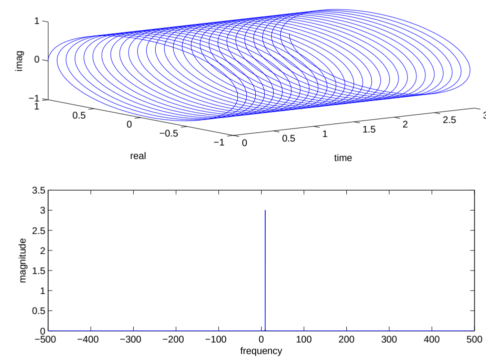
```
Ts = 0.001;
t_max = 3;
t = 0:Ts:t_max;
x = zeros(size(t));
x(1) = 1/Ts;
plottf(x,Ts);
```



It is encouraging to see that the frequency magnitude response matches that of $\mathcal{F}\{\delta(t)\} = 1$ (over the frequency range that is plotted by `plottf`).

(b) The complex exponential and its `plottf`-approximated Fourier transform are (for $t_{\max} = 3$):

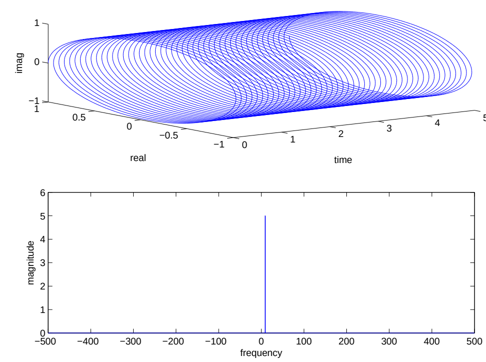
```
Ts = 0.001;
t_max = 3;
t = 0:Ts:t_max;
f_o = 9;
x = exp(j*2*pi*f_o*t);
plottf(x,Ts);
```



While the basic shape of the frequency magnitude response matches that of $\mathcal{F}\{\exp(j2\pi f_o t)\} = \delta(f - f_o)$, it is impossible to actually plot $\delta(f - f_o)$ due to its infinite height.

(c) The complex exponential and its `plottf`-approximated Fourier transform are (for $t_{\max} = 5$):

```
Ts = 0.001;
t_max = 5;
t = 0:Ts:t_max;
f_o = 9;
x = exp(j*2*pi*f_o*t);
plottf(x,Ts);
```



The difference between the case $t_{\max} = 5$ and $t_{\max} = 3$ is that the height of the spike is now 5 rather than 3. It is interesting that changing the *length* of the time-domain waveform changes the *amplitude* in the frequency domain. This behavior can be traced back to equations (1)-(2) on the homework, which show how the FT is approximated by `plottf`: as $t_{\max}$ increases, so does the number of samples N, which causes the sum in (2) to grow proportionally.