

Introduction:

Goal: Transmit a message from one location to another.

When message is . . .

continuous waveform \rightarrow analog comm (e.g., FM radio),

sequence of numbers \rightarrow digital comm (e.g., mp3 file),

though the sequence of numbers might represent a continuous waveform (as in the case of mp3 audio).

Typical communication media:

twisted pair wire (e.g., telephone_A)

coaxial cable (e.g., TV_{A,D}, data_D)

fiber optic cable (e.g., ethernet_D)

EM waves (e.g., cellular phones_{A,D}, WiFi_D, TV_{A,D})

water waves (e.g., underwater network_{A,D})

power lines_{A,D}

compact disc_D

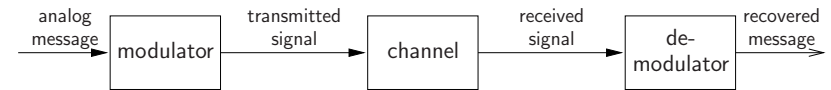
hard drive_D

magnetic tape_{A,D}

where A = analog and D = digital.

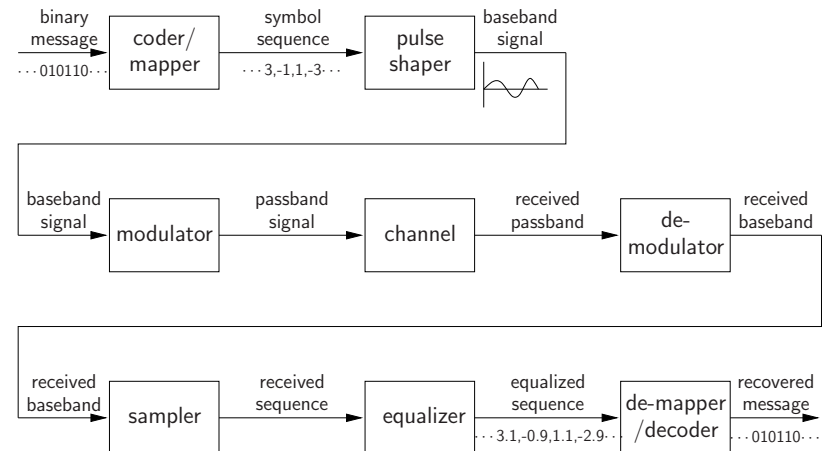
Note that, whether the message signal is discrete-time or continuous-time, the transmitted signal is continuous-time!

Analog Communication:



- Perfect recovery is impossible in the presence of noise!

Digital Communication:

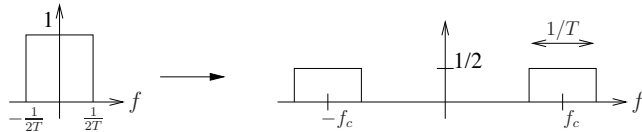


- A digital message is converted to an analog message coding and pulse-shaping, and then transmitted using analog modulation. To recover the message, the received signal is demodulated, sampled, and digitally processed.
- Perfect recovery is possible even in the presence of noise!

Preview of Comm System Components:

Modulator:

- Translates “baseband” analog signal to “passband”:



where f_c is the “carrier frequency.”

- There are two principal motivations for doing this:
 1. Often we want to communicate several signals simultaneously (e.g., TV, radio, voice). It's difficult or impossible to do this if they overlap in frequency!
 2. Wireless EM transmission/reception is much easier at higher frequencies, since need antenna length $> \frac{\lambda}{10}$. ($\lambda = \frac{c}{f_c}$ is wavelength and $c=3e8$ m/s speed of light.)

system	transmission band	$\lambda/10$
VHF (TV)	30–300 MHz	1–0.1 m
UHF (TV)	0.3–3 GHz	10–1 cm
cellular	824–960 MHz	3 cm
WiFi	2.4 GHz	1 cm

Notice that practical antenna length determines where different signal types can be transmitted.

Coder/Mapper:

- Coder transforms sequence of message bits into an error-resilient sequence of coded bits.
- Mapper transforms coded bits into discrete “symbols.”

Ex: If the “symbol alphabet” is $\{-3, -1, 1, 3\}$ and the symbol mapping is

bits	symbol
00	3
01	-1
10	1
11	-3

, then ASCII text would

be transmitted via

letter	ASCII code				symbol sequence			
a	01	10	00	01	-1	1	-3	-1
b	01	10	00	10	-1	1	-3	1
c	01	10	00	11	-1	1	-3	3
d	01	10	01	00	-1	1	-1	-3
⋮								

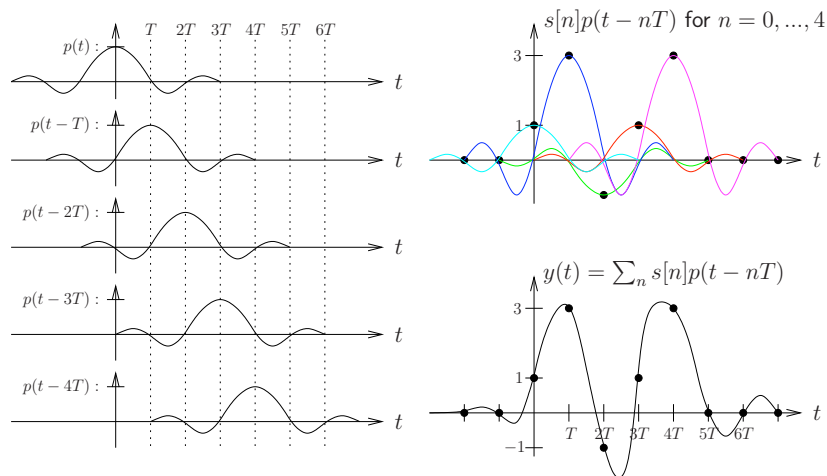
Pulse Shaper:

- Converts symbol sequence into a continuous waveform.
- In linear modulation schemes, the time- n symbol $s[n]$ scales a nT -delayed version of pulse $p(t)$:

$$y(t) = \sum_n s[n]p(t - nT) \quad \text{"baseband signal"}$$

$$T = \text{"symbol period"}$$

Ex: Say symbol sequence is $[1, 3, -1, 1, 3]$. Then



Preliminaries (Ch.2):

Fourier Transform (FT):

Definition:

$$W(f) = \int_{-\infty}^{\infty} w(t)e^{-j2\pi ft} dt = \mathcal{F}\{w(t)\}$$

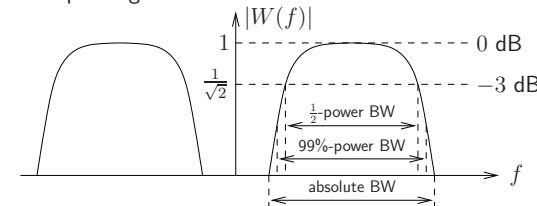
$$w(t) = \int_{-\infty}^{\infty} W(f)e^{j2\pi ft} df = \mathcal{F}^{-1}\{W(f)\}.$$

Properties:

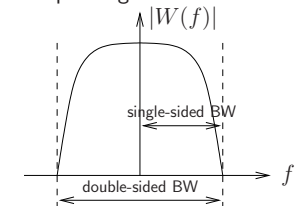
- Linearity: $\mathcal{F}\{c_1w_1(t) + c_2w_2(t)\} = c_1W_1(f) + c_2W_2(f)$.
- Real-valued $w(t) \Rightarrow \begin{cases} \text{conjugate symmetric } W(f) \\ |W(f)| \text{ symmetric around } f = 0. \end{cases}$

"Bandwidth":

bandpass signal:

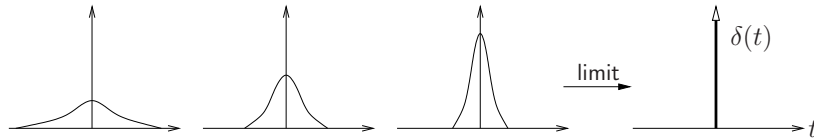


lowpass signal:



Dirac Delta (or “continuous impulse”) $\delta(\cdot)$:

An infinitely tall and thin waveform *with unit area*:



that’s often used to “kick” a system and see how it responds.

Key properties:

1. Sifting: $\int_{-\infty}^{\infty} w(t)\delta(t - q)dt = w(q)$.

2. Time-domain impulse $\delta(t)$ has a flat spectrum:

$$\mathcal{F}\{\delta(t)\} = \int_{-\infty}^{\infty} \delta(t)e^{-j2\pi ft} dt = 1 \text{ (for all } f\text{)}.$$

3. Freq-domain impulse $\delta(f)$ corresponds to a DC waveform:

$$\mathcal{F}^{-1}\{\delta(f)\} = \int_{-\infty}^{\infty} \delta(f)e^{j2\pi ft} df = 1 \text{ (for all } t\text{)}.$$

Frequency-Domain Representation of Sinusoids:

Notice from the sifting property that

$$\mathcal{F}^{-1}\{\delta(f - f_o)\} = \int_{-\infty}^{\infty} \delta(f - f_o)e^{j2\pi ft} df = e^{j2\pi f_o t}.$$

Thus, Euler’s equations

$$\cos(2\pi f_o t) = \frac{1}{2}e^{j2\pi f_o t} + \frac{1}{2}e^{-j2\pi f_o t}$$

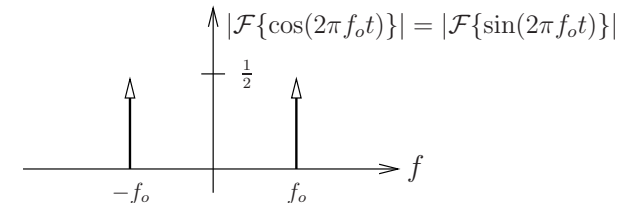
$$\sin(2\pi f_o t) = \frac{1}{2j}e^{j2\pi f_o t} - \frac{1}{2j}e^{-j2\pi f_o t}$$

and the Fourier transform pair $e^{j2\pi f_o t} \leftrightarrow \delta(f - f_o)$ imply that

$$\mathcal{F}\{\cos(2\pi f_o t)\} = \frac{1}{2}\delta(f - f_o) + \frac{1}{2}\delta(f + f_o)$$

$$\mathcal{F}\{\sin(2\pi f_o t)\} = \frac{1}{2j}\delta(f - f_o) - \frac{1}{2j}\delta(f + f_o).$$

Often we draw this as



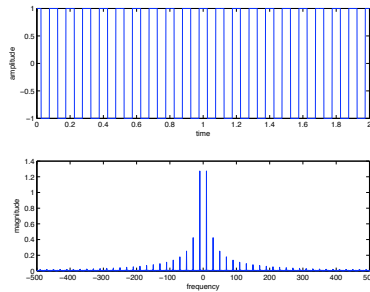
Frequency Domain via MATLAB:

Fourier transform requires evaluation of an integral. What do we do if we can't define/solve the integral?

1. Generate (rate- $\frac{1}{T_s}$) sampled signal in MATLAB.
2. Plot magnitude of Discrete Fourier Transform (DFT) using `plottf.m` (from course webpage).

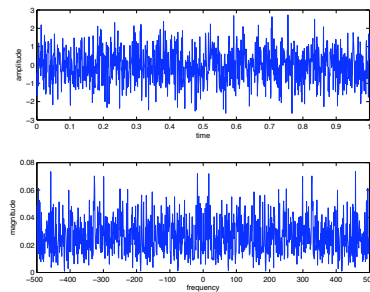
Square-wave example:

```
f = 10;
t_max = 2;
Ts = 1/1000;
t = 0:Ts:t_max;
x = sign(cos(2*pi*f*t));
plottf(x,Ts);
```



Noise-wave example:

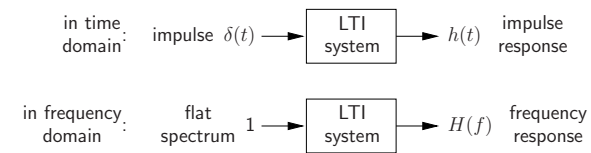
```
t_max = 1;
Ts = 1/1000;
x = randn(1,t_max/Ts);
plottf(x,Ts);
```



Notice that `plottf.m` only plots frequencies $f \in [-\frac{1}{2T_s}, \frac{1}{2T_s}]$.

Linear Time-Invariant (LTI) Systems:

An LTI system can be described by either its "impulse response" $h(t)$ or its "frequency response" $H(f) = \mathcal{F}\{h(t)\}$.



Input/output relationships:

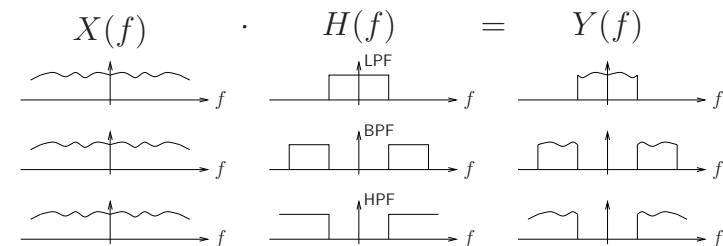
- Time-domain: Convolution with impulse response $h(t)$

$$x(t) \rightarrow h(t) \rightarrow y(t) \quad y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(t - \tau)x(\tau)d\tau$$
- Freq-domain: Multiplication with freq response $H(f)$

$$X(f) \rightarrow H(f) \rightarrow Y(f) \quad Y(f) = H(f)X(f)$$

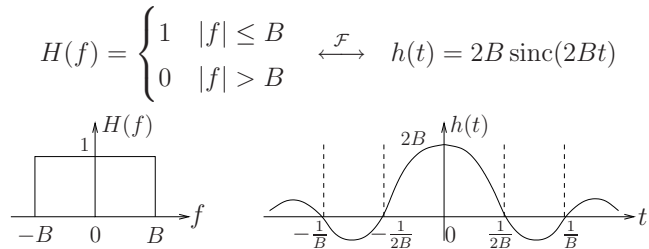
Linear Filtering:

Freq-domain illustration of LPF, BPF, and HPF:

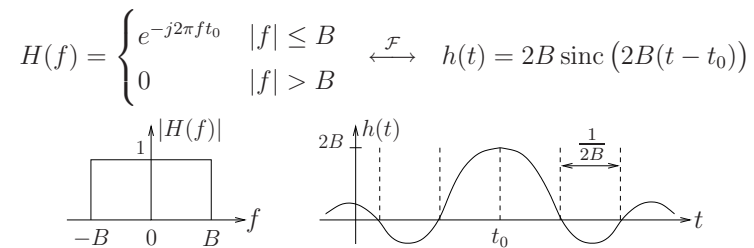


Lowpass Filters:

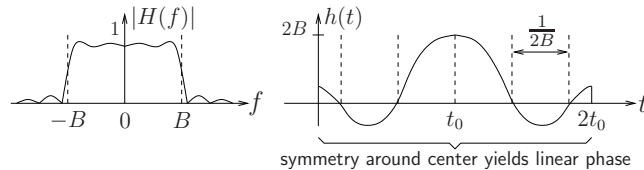
Ideal non-causal LPF (using $\text{sinc}(x) := \frac{\sin(\pi x)}{\pi x}$):



Ideal LPF with group-delay t_o :



A causal linear-phase LPF with group-delay t_o :

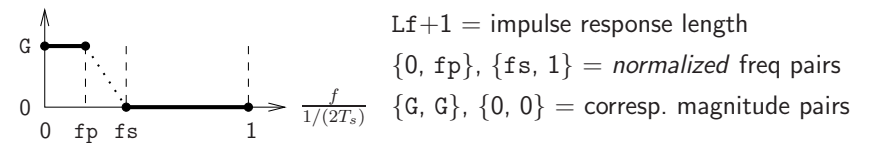


but MATLAB can give better causal linear-phase LPFs...

In MATLAB, generate $\frac{1}{T_s}$ -sampled LPF impulse response via

$$h = \text{firls}(\text{Lf}, [0, \text{fp}, \text{fs}, 1], [\text{G}, \text{G}, 0, 0]) / T_s;$$

where...

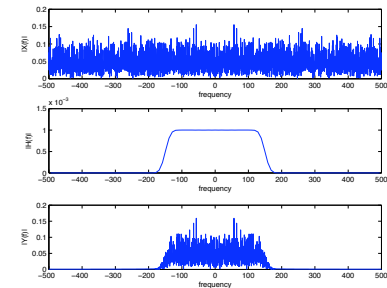


The commands `firpm` and `fir2` have the same interface, but yield slightly different results (often worse for our apps).

In MATLAB, perform filtering on $\frac{1}{T_s}$ -sampled signal x via

$$y = T_s * \text{filter}(h, 1, x); \text{ or } y = T_s * \text{conv}(h, x);$$

```
t_max = 3; Ts = 1/1000;
x = randn(1, t_max/Ts);
h = firls(100, [0, 0.2, 0.4, 1], [1, 1, 0, 0])/Ts;
y = Ts*filter(h, 1, x);
subplot(3, 1, 1);
plottf(x, Ts, 'f');
ylabel('|X(f)|')
subplot(3, 1, 2);
plottf(h, Ts, 'f');
ylabel('|H(f)|')
subplot(3, 1, 3);
plottf(y, Ts, 'f');
ylabel('|Y(f)|')
```



Important: The routines `firls`, `firpm`, `fir2` generate *causal* linear-phase filters with group delay $= \frac{\text{Lf}}{2}$ samples. Thus, the filtered output y will be delayed by $\frac{\text{Lf}}{2}$ samples relative to x .