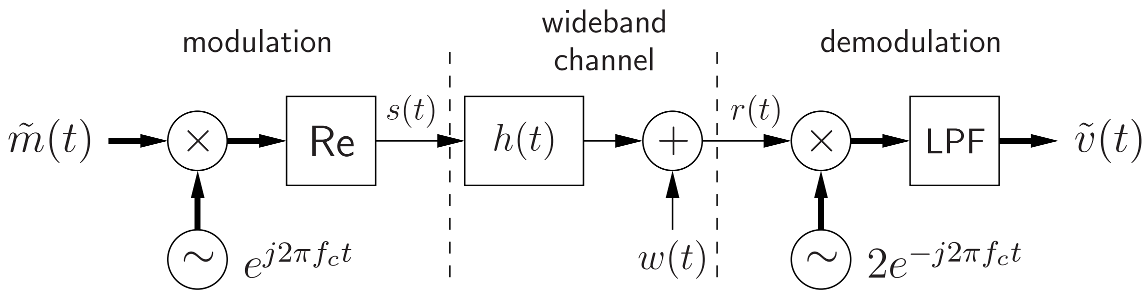
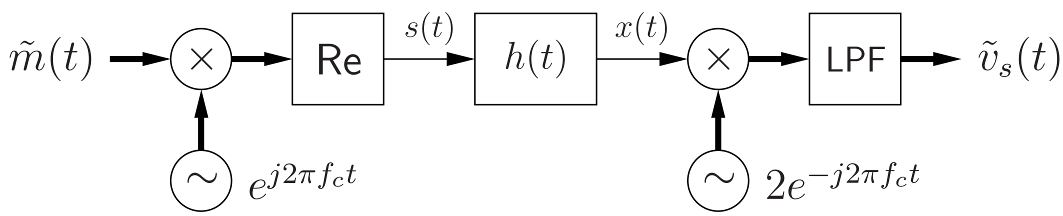
Complex-Baseband Equivalent Channel:

Linear communication schemes (e.g., AM, QAM, VSB) can all be represented (using complex-baseband mod/demod) as:



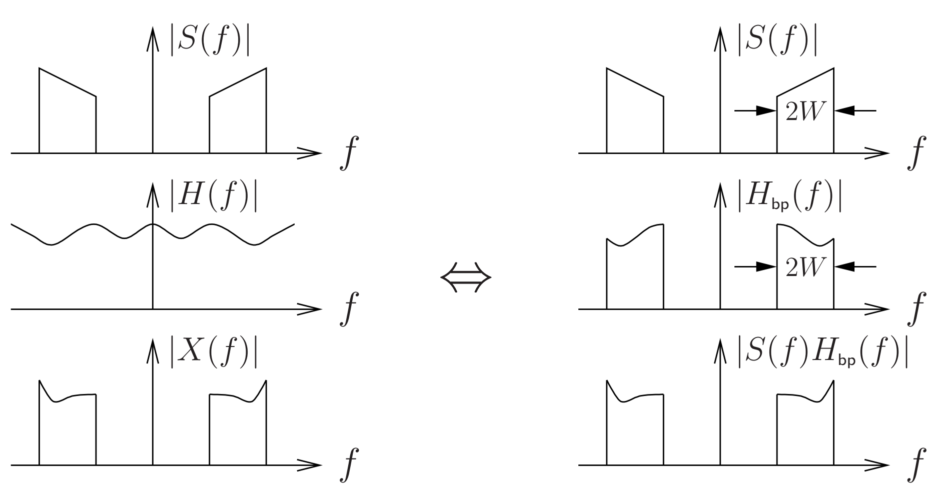
It turns out that this diagram can be greatly simplified...

First, consider the signal path on its own:



Since $s(t)$ is a bandpass signal, we can replace the *wideband* channel response $h(t)$ with its *bandpass equivalent* $h_{\text{bp}}(t)$:

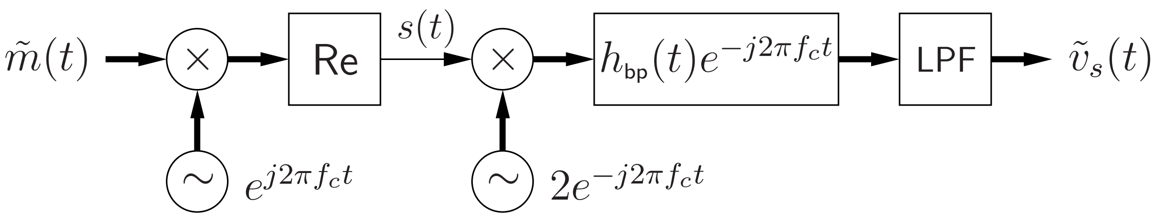
...because filtering $s(t)$ with $h(t)$ is equivalent to filtering $s(t)$ with $h_{\text{bp}}(t)$:



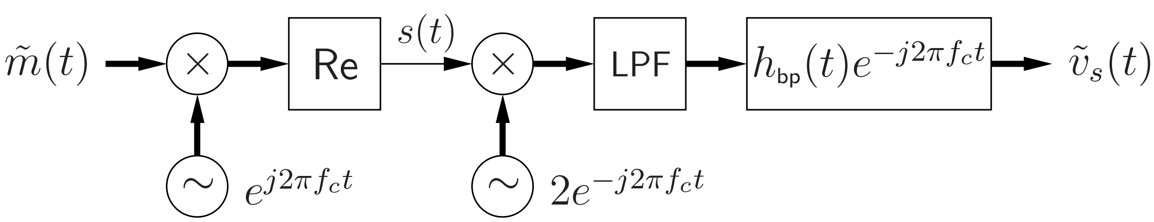
Then, notice that

$$\begin{aligned}
\big[s(t) * h_{\mathsf{bp}}(t)\big] 2e^{-j2\pi f_c t} &= \int s(\tau) h_{\mathsf{bp}}(t-\tau) d\tau \cdot 2e^{-j2\pi f_c t} \\
&= \int s(\tau) 2e^{-j2\pi f_c \tau} h_{\mathsf{bp}}(t-\tau) e^{-j2\pi f_c (t-\tau)} d\tau \\
&= \big[s(t) 2e^{-j2\pi f_c t}\big] * \big[h_{\mathsf{bp}}(t) e^{-j2\pi f_c t}\big],
\end{aligned}$$

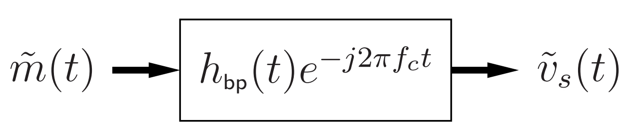
which means we can rewrite the block diagram as



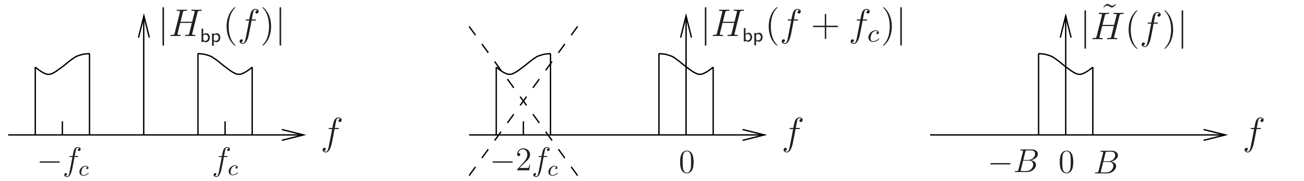
We can now reverse the order of the LPF and $h_{\mathsf{bp}}(t)e^{-j2\pi f_c t}$ (since both are LTI systems), giving



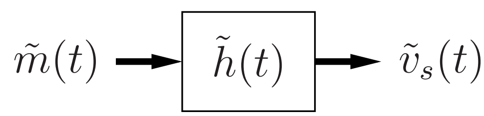
Since mod/demod is transparent (with synched oscillators), it can be removed, simplifying the block diagram to



Now, since $\tilde{m}(t)$ is bandlimited to W Hz, there is no need to model the left component of $H_{\text{bp}}(f+f_c) = \mathcal{F}\{h_{\text{bp}}(t)e^{-j2\pi f_c t}\}$:

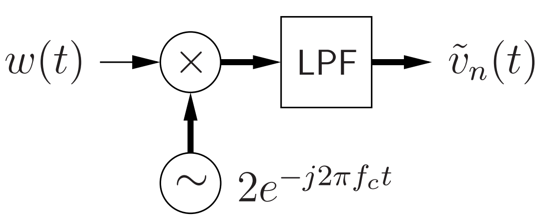


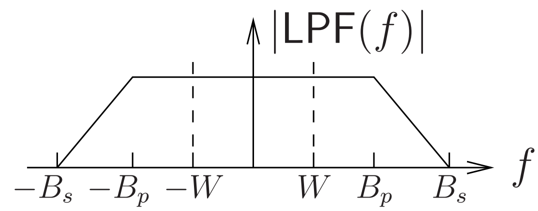
Replacing $h_{\text{bp}}(t)e^{-j2\pi f_c t}$ with the *complex-baseband response* $\tilde{h}(t)$ gives the "complex-baseband equivalent" signal path:



The spectrums above show that $h_{\text{bp}}(t) = \text{Re}\{\tilde{h}(t) \cdot 2e^{j2\pi f_c t}\}$.

Next consider the noise path on it's own:

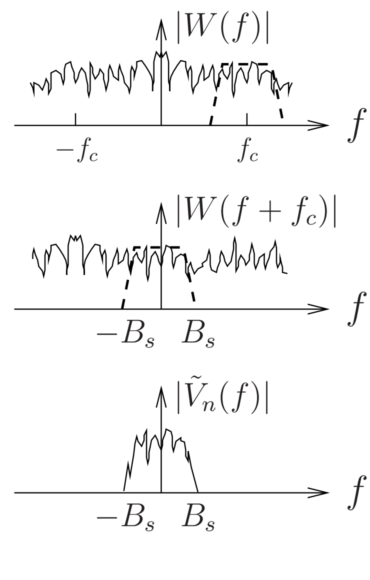




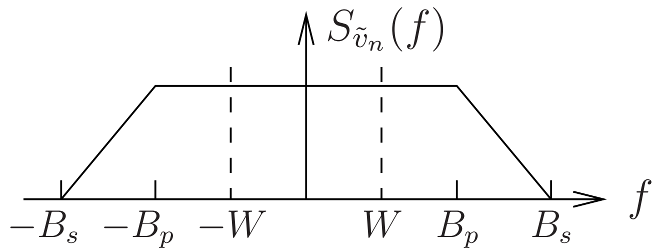
From the diagram, $\tilde{v}_n(t)$ is a baseband version of the bandpass noise spectrum that occupies the frequency range

$$f \in [f_c - B_s, f_c + B_s].$$

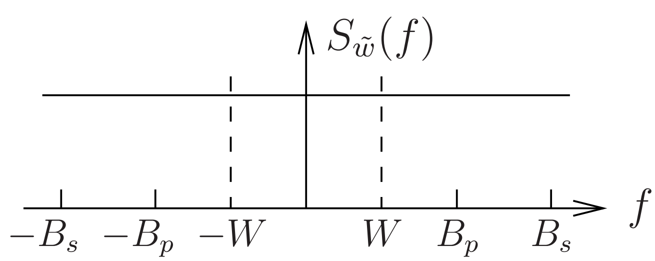
Since $\tilde{v}_n(t)$ is complex-valued, $\tilde{V}_n(f)$ is non-symmetric.



Say that $w(t)$ is real-valued white noise with power spectral density (PSD) $S_w(f) = N_0$. Since $S_w(f)$ is constant over all f, the PSD of the complex noise $\tilde{v}_n(t)$ will be constant over the LPF passband, i.e., $f \in [-B_p, B_p]$:

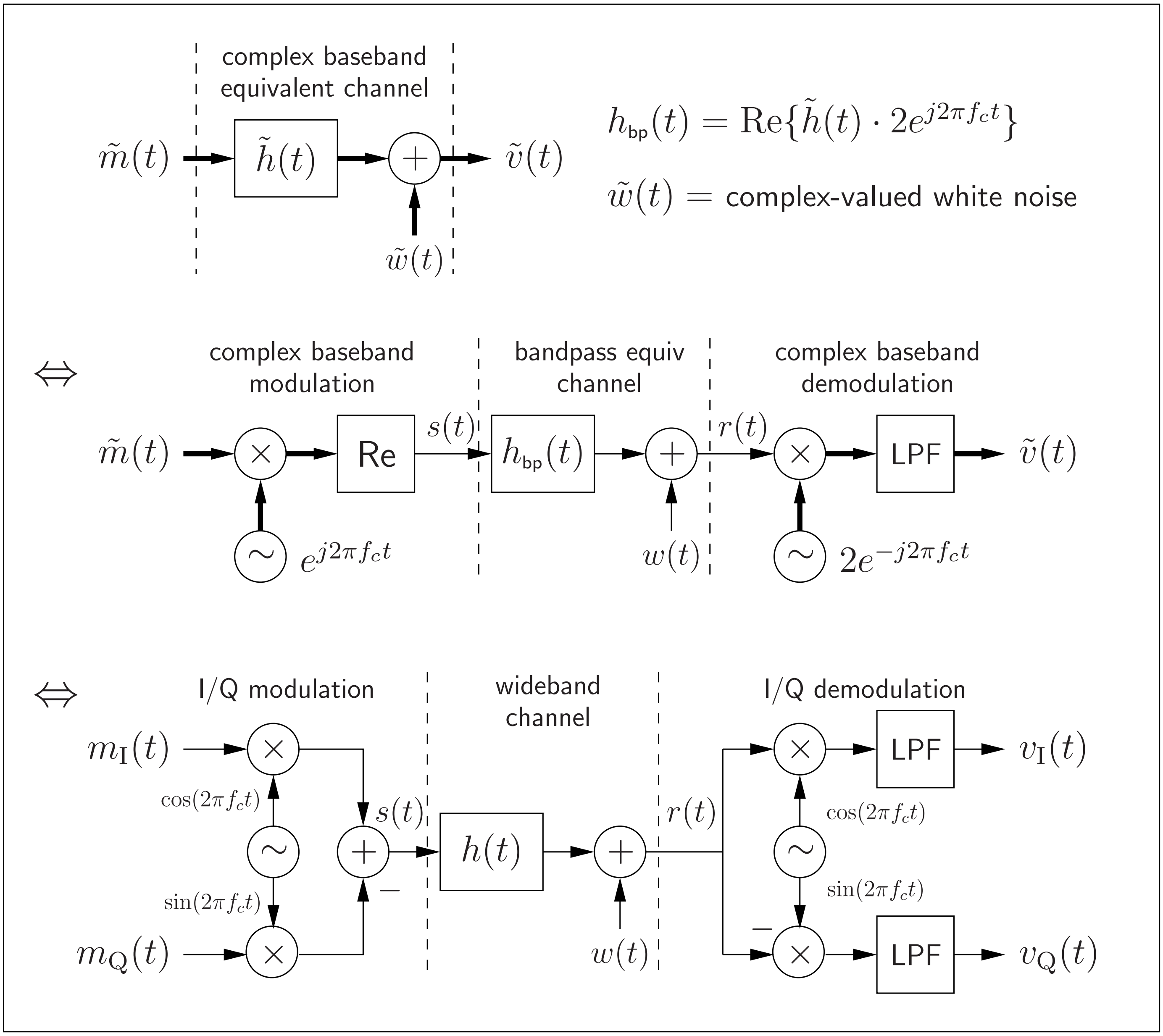


A well-designed communications receiver will suppress all energy outside the signal bandwidth W, since it is purely noise. Given that the noise spectrum outside $f \in [-W, W]$ will get totally suppressed, *it doesn't matter how we model it!* Thus, we choose to replace the lowpass complex noise $\tilde{v}_n(t)$ with something simpler to describe: *white* complex noise $\tilde{w}(t)$ with PSD $S_{\tilde{w}}(f) = N_0$:



We'll refer to $\tilde{w}(t)$ as "*complex baseband equivalent*" noise.

Putting the signal and noise paths together, we arrive at the *complex baseband equivalent channel model*:



The diagrams above should convince you of the utility of the complex-baseband representation in simplifying the system model!