ECE-501 **Intro to Communications** **Autumn 2007**

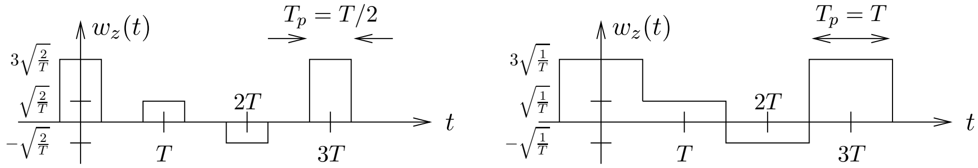
Midterm Examination #2 Nov. 21, 2007

MIDTERM #2 SOLUTIONS

1. (a) With digital modulation, the transmitted signal $w_z(t)$ obeys

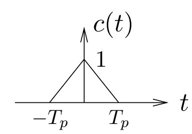
$$w_z(t) = \sum_n s[n]p(t - nT).$$

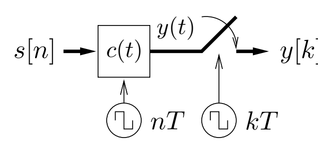
The values $\big[s[0], s[1], s[2], s[3]\big] = [3, 1, -1, 3]$, yield $w_z(t)$ illustrated below for $T_p = T/2$ (left) and $T_p = T$ (right).



(b) Recall that convolving two symmetric rectangles $p(t)$ of width T_p yields a symmetric triangle $c(t)$ of width $2T_p$ and height $\int_{-\infty}^{\infty} |p(t)|^2 dt$. Here, $\int_{-\infty}^{\infty} |p(t)|^2 dt = \int_{-T_p/2}^{T_p/2} \big|\frac{1}{\sqrt{T_p}}\big|^2 dt = 1$. Thus, we get the following:

$$c(t) = \begin{cases} \frac{t+T_p}{T_p} & t \in [-T_p, 0) \\ \frac{T_p - t}{T_p} & t \in [0, T_p) \\ 0 & \text{else} \end{cases}$$

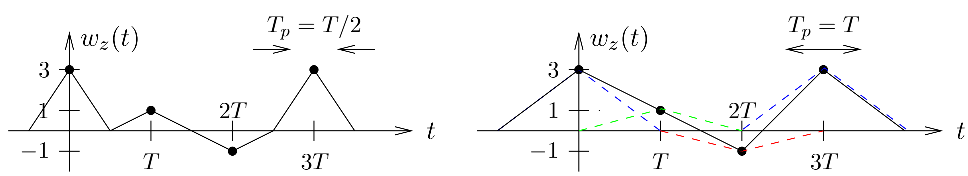




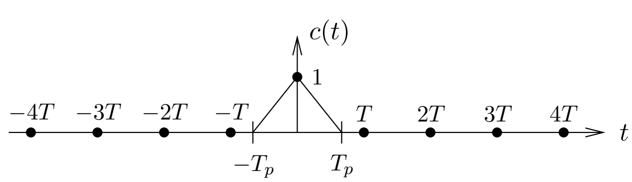
(c) With a noiseless trivial channel, the received signal obeys

$$y(t) = \sum_n s[n]c(t - nT).$$

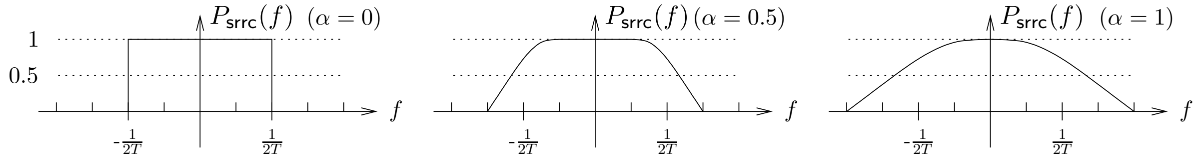
The values $\big[s[0], s[1], s[2], s[3]\big] = [3, 1, -1, 3]$, yield $y(t)$ illustrated below for $T_p = T/2$ (left) and $T_p = T$ (right). The individual contributions of each symbol are depicted using dashed lines.



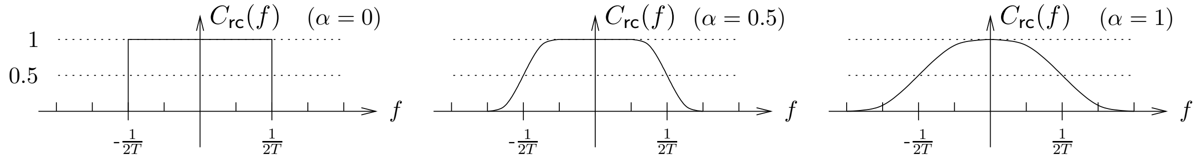
(d) The Nyquist criterion can be stated as $c(mT) = \begin{cases} 1 & m = 0 \\ 0 & m \neq 0 \end{cases}$ for integer m. As evident from the figure below, the Nyquist criterion is satisfied for any $T_p \in (0, T]$:



2. (a) The SRRC frequency response $P_{\mathsf{srrc}}(f)$ is sketched below for various α.



(b) The RC frequency response $C_{\mathsf{rc}}(f)$ is sketched below for various α.



(c) From the homework, we know $S_y(f) = |P_{\mathsf{srrc}}(f)|^2 S_n(f)$. Since $P_{\mathsf{srrc}}(f) \in \mathbb{R}$, we have

$$\begin{aligned} S_y(f) &= P_{\mathsf{srrc}}(f)^2 N_0 \\ &= \boxed{C_{\mathsf{rc}}(f) N_0}. \end{aligned}$$

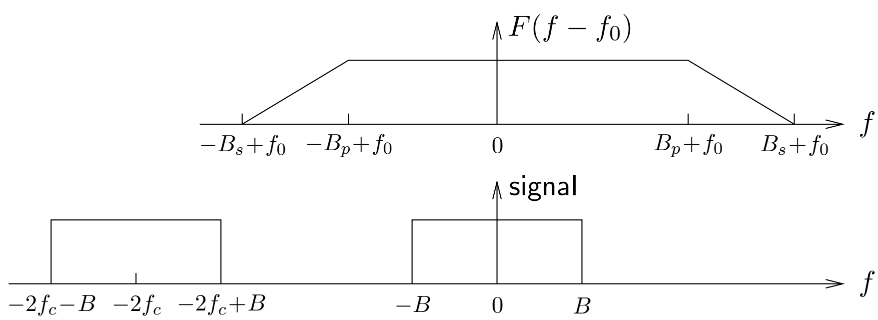
3. (a) Because $2e^{-j2\pi(f_c+f_0)t} = 2e^{-j2\pi f_c t} \cdot e^{-j2\pi f_0 t}$, we just need to put $\boxed{e^{-j2\pi f_0 t}}$ in the box.

(b) The diagram on the left implies

$$\begin{aligned} x(t) &= [u(t)e^{-j2\pi f_0 t}] * f(t) \\ &= \int_{-\infty}^{\infty} u(\tau)e^{-j2\pi f_0 \tau} f(t-\tau)d\tau \\ &= \int_{-\infty}^{\infty} u(\tau)f(t-\tau)e^{j2\pi f_0(t-\tau)}d\tau \cdot e^{-j2\pi f_0 t} \\ &= \Big[u(t) * [f(t)e^{j2\pi f_0 t}]\Big] e^{-j2\pi f_0 t} \end{aligned}$$

which can be interpreted as "$u(t)$ gets convolved with $f(t)e^{j2\pi f_0 t}$, and the result is modulated by $e^{-j2\pi f_0 t}$." Thus, the impulse response $\boxed{f(t)e^{j2\pi f_0 t}}$ goes in the box.

(c) Since $\mathcal{F}\{f(t)e^{j2\pi f_0 t}\} = F(f - f_0)$, the modulation acts to shift the passband of the LPF right by f_0 Hz:



The effect of this modulation on up/down-conversion can be understood by recalling the two key roles of the LPF: (i) suppressing the undesired signal component that exists at $f \in [-2f_c - B, -2f_c + B]$ Hz and (ii) preserving the desired signal component that exists at $f \in [-B, B]$ Hz.

i. To suppress the unwanted component, we need $-2f_c + B \leq -B_s + f_0$, or equivalently $\boxed{f_0 \geq B_s + B - 2f_c}$.

ii. To preserve the desired component, we need $-B_p + f_0 \leq -B$ and $B_p + f_0 \geq B$, or equivalently $-B_p + B \leq f_0 \leq B_p - B$ or equivalently $\boxed{|f_0| \leq B_p - B}$.

4. (a) $h(t)$ can be replaced with $h_p(t)$ because these filters are identical at all frequencies occupied by the input signal (i.e., $f \in [-f_c - B, -f_c + B] \cup [f_c - B, f_c + B]$).

   (b) As in problem 3(a), we can write $2e^{-j2\pi(f_c+f_0)t} = 2e^{-j2\pi f_c t} \cdot e^{-j2\pi f_0 t}$.

   (c) Problem 3(b) confirms that this is the correct way to reverse the order of filtering and modulation.

   (d) We can always swap the ordering of linear time-invariant operations.

   (e) Problem 3(c) shows that up/down-conversion is transparent even when the LPF is modulated by f_0 Hz, provided that $|f_0| \leq B_p - B$ and $f_0 \geq B_s + B - 2f_c$.

   (f) $h_p(t)e^{-j2\pi f_c t}$ can be replaced with $h_z(t)$ because these filters are identical at all frequencies occupied by the input signal (i.e., $f \in [-B, B]$).