Midterm Exam $\#2$ Nov. 21, 2007

MIDTERM EXAMINATION #2

Name:

Instructions:

- Do not turn over this cover page until instructed to do so.
- You will have 48 minutes to complete this exam.
- You are allowed to consult the letter-sized piece of paper which you have prepared beforehand. You are not allowed to consult any other books, notes, turkeys, pheasants, or fowl.
- Please write clearly and include sufficient explanation with all of your answers.
- If you write on the backs of the pages, indicate this so the grader does not miss your work.
- Do not unstaple the test pages.

1. (20 pts)

Consider a digital communication system with a trivial noiseless channel that uses identical pulse shapes $p(t) = g(t)$ at the transmitter and receiver. In particular, consider the *square* T_p -wide pulse illustrated below with a symbol period of T seconds.

$$
p(t) = g(t) = \begin{cases} \frac{1}{\sqrt{T_p}} & t \in \left[-\frac{T_p}{2}, \frac{T_p}{2}\right) \\ 0 & \text{else} \end{cases} \qquad \qquad \frac{1}{\sqrt{T_p}} \begin{cases} p(t) & s[n] \rightarrow p(t) \rightarrow \infty \\ \frac{1}{\sqrt{T_p}} & t \rightarrow \infty \end{cases} \qquad \qquad \frac{1}{\sqrt{T_p}} \begin{cases} p(t) & s[n] \rightarrow p(t) \rightarrow \infty \\ \frac{1}{\sqrt{T_p}} & t \rightarrow \infty \end{cases} \qquad \qquad \frac{1}{\sqrt{T_p}} \begin{cases} p(t) & \text{if } t \rightarrow \infty \\ \frac{1}{\sqrt{T_p}} & t \rightarrow \infty \end{cases}
$$

(a) For symbol sequence $[s[0], s[1], s[2], s[3] = [3, 1, -1, 3]$, where $s[n]$ is otherwise zero, sketch the transmitted signal $w_z(t)$ for two cases: $T_p = T/2$ and $T_p = T$.

(b) Sketch the combined pulse $c(t) = p(t) * g(t)$ for arbitrary T_p . (*Hint*: To save time, try to construct $c(t)$ visually.)

(c) For symbol sequence $[s[0], s[1], s[2], s[3] = [3, 1, -1, 3]$, where $s[n]$ is otherwise zero, sketch the received signal $y(t)$ for two cases: $T_p = T/2$ and $T_p = T$.

(d) For what values of T_p will the Nyquist criterion for ISI prevention be satisfied? (*Hint*: Consider the time-domain criterion, and examine $c(mT)$ for integers m.)

2. (15 pts)

For this problem, assume a symbol period of T seconds.

(a) Sketch the frequency response of square-root raised-cosine (SRRC) pulse, i.e., $P_{\text{src}}(f)$, for each $\alpha \in \{0, 0.5, 1\}.$

(b) Sketch the frequency response of the raised-cosine (RC) pulse, i.e., $C_{\rm rc}(f)$, for each $\alpha \in \{0, 0.5, 1\}.$

(c) Consider feeding white noise $n(t)$ with power spectral density (PSD) $S_n(f) = N_0$ into a linear filter whose frequency response is $P_{\text{src}}(f)$. What is $S_y(f)$, the PSD of the output signal $y(t)$?

$$
n(t) \longrightarrow \boxed{P_{\text{src}}(f)} \longrightarrow y(t)
$$

3. (15 pts)

The following subproblems yield tools that will be useful for Problem 4.

(a) To make the following two block diagrams equivalent, what signal goes in the box?

(b) To make the following two block diagrams equivalent, what impulse response goes in the box?

$$
u(t) \rightarrow \begin{array}{|c|c|c|c|c|c|c|} \hline \begin{array}{ccc} \bullet & & \bullet & \\ \bullet & & \bullet & \\ \bullet & & \bullet & \\ \bullet & & & \bullet \\ \hline \end{array} & e^{-j2\pi f_0 t} & & & \bullet \\ \hline \end{array} \qquad \Leftrightarrow \qquad u(t) \rightarrow \begin{array}{|c|c|c|c|} \hline ? & & \bullet & \\ \bullet & & & \bullet \\ \bullet & & & \bullet \\ \hline \end{array} & x(t)
$$

(c) Here we depict the standard up/down-conversion process, using LPF $f(t)$ and input signal $w_z(t)$ with single-sided bandwidth B Hz:

$$
w_z(t) \longrightarrow \infty
$$
 Re \longrightarrow Re $f(t)$ $x(t)$ $f(f)$
\n
$$
\sim e^{j2\pi f_c t}
$$
 $\sim 2e^{-j2\pi f_c t}$ \sim $\frac{F(f)}{-B_s - B_p - B}$ $B B_p B_s$

We know that the system above gives perfect reconstruction (i.e., $x(t) = w_z(t)$). Now consider the same system, but with a LPF that is modulated by f_0 Hz:

$$
w_z(t) \longrightarrow \infty
$$
 Re
\n
$$
\longrightarrow
$$
 Re
\n
$$
\longrightarrow
$$

$$
f(t)e^{j2\pi f_0 t} \longrightarrow x(t)
$$

\n
$$
\longrightarrow
$$

$$
e^{j2\pi f_c t}
$$

$$
\longrightarrow
$$

$$
2e^{-j2\pi f_c t}
$$

What range of f_0 can be tolerated while still achieving perfect reconstruction? Your answer should involve $\{B, B_p, B_s, f_c\}$. (*Hint*: Consider the two key responsibilities of the LPF.)

4. (20 pts)

Here we will derive the (noiseless) complex-baseband equivalent channel model for the case where the receiver oscillator has a frequency offset of f_0 Hz. As usual, we start with the following block diagram, where the signal $w_z(t)$ has a single-sided bandwidth of B Hz.

In all parts below, try to keep your responses to one sentence, and write them in the boxes provided. Your answers may refer to previous problems on the midterm.

(a) Explain why the block diagram can be rewritten as follows.

(b) Explain why the block diagram can be rewritten as follows.

(c) Explain why the block diagram can be rewritten as follows.

(d) Explain why the block diagram can be rewritten as follows.

(e) Explain why the block diagram can be rewritten as follows for a range of f_0 , and specify that range.

$$
w_z(t) \longrightarrow h_p(t)e^{-j2\pi f_c t} \longrightarrow (\times \longrightarrow x_s(t))
$$

$$
\longleftrightarrow e^{-j2\pi f_0 t}
$$

(f) Explain why the block diagram can be rewritten as follows.

$$
w_z(t) \longrightarrow h_z(t) \longrightarrow (\times) \longrightarrow x_s(t)
$$

$$
\longleftrightarrow e^{-j2\pi f_0 t}
$$