HOMEWORK ASSIGNMENT #8

Due Fri. Mar. 7, 2008 (in class)

1. Say that your project partner suggests to use the symbol alphabet $\{-\Delta, 0, 2\Delta\}$ to send data in a communication system whose outputs are well modeled by $y[n] = a[n] + e[n]$, where $a[n]$ is the data symbol and $e[n]$ is real-valued Gaussian error with zero-mean and variance σ_e^2. Assuming a data sequence such that the alphabet entries are chosen with equal probability...

   (a) Give an expression for the symbol power $\mathrm{E}\{|a[n]|^2\}$. (*Hint:* Notice that $a[n] = -\Delta$ with probability 1/3, $a[n] = 0$ with probability 1/3, and $a[n] = 2\Delta$ with probability 1/3. So, what is the average value of $|a[n]|^2$?)

   (b) Give an expression for the symbol error rate (SER) of nearest-element decisions in terms of Δ, σ_e^2, and the Q function.

   (c) Using the answer from part (a), rewrite the SER expression from (b) so that it depends on the $\mathrm{E}\{|a[n]|^2\}$ instead of Δ.

   (d) Based on your answer from part (c), how does the SER of the proposed scheme compare to that of 3-PAM, 4-PAM, and 5-PAM for the same $\mathrm{E}\{|a[n]|^2\}/\sigma_e^2$? To investigate this, plot the SER versus $\mathrm{E}\{|a[n]|^2\}/\sigma_e^2 \in [1, 100]$ on a log-log scale. (*Hint:* Generate the values of $\mathrm{E}\{|a[n]|^2\}/\sigma_e^2$ using `logspace` and plot using `loglog`.)

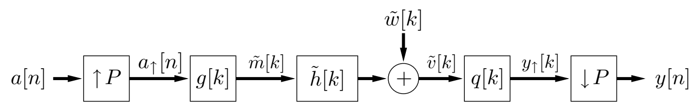
2. For the 2-PAM, 4-PAM, and 8-PAM alphabets, write a MATLAB routine that makes nearest-element decisions from the observed samples $y[n] = a[n] + e[n]$ and returns the calucated symbol error rate (by counting the number of decision errors) as well as the theoretical error rate (via `erfc`). Here, $\{a[n]\}_{n=0}^{N-1}$ are symbols generated using the `pam` command and $\{e[n]\}_{n=0}^{N-1}$ are Gaussian errors generated using[1] `randn`. In all cases, use $\sigma_a^2 = 1$, $\sigma_e^2 = 0.1$, and $N = 1 \times 10^6$. Present your results in table form:

	2-PAM	4-PAM	8-PAM
experimental SER	•	•	•
theoretical SER	•	•	•

   Hint: To avoid slow MATLAB code, I suggest to make decisions simultaneously on the entire *vector* of outputs, rather than on each output $y[n]$ separately (in a `for` loop). This can be done using the `round` command (in conjunction with `min` and `max` to handle the edge points).

[1]The command `randn` produces zero-mean white Gaussian noise with unit variance. For variance-σ_e^2 noise, simply scale the output of `randn` by $\sqrt{\sigma_e^2}$.

3. While the previous problem used the simplified model $y[n] = a[n] + e[n]$, we now take $y[n]$ to be the output of the digital communication system illustrated below, which experiences a noisy but otherwise trivial channel (i.e., $\tilde{h}[k] = \delta[k]$). Assume oversampling factor $P = 2$, SRRC pulses



with parameter $\alpha = 0.5$ that are truncated to the interval $[-2T, 2T]$, a unit-variance BPSK (i.e., 2-PAM) symbol sequence of length $N = 1 \times 10^4$, and white complex-baseband[2] noise $n_z[k]$ with variance $\sigma_w^2 = 0.5$.

(a) Modify your constellation-diagram code from the previous homework assignment to include the complex-baseband noise $n_z[k]$, and plot the constellation diagram from $y_\uparrow[k]$.

(b) Incorporating the code you wrote in the previous problem, calculate the experimental SER (based on symbol decisions from $y[n]$) as well as the theoretical SER. When calculating the theoretical SER, use $\sigma_w^2/2$ for the variance of the real-valued noise component (in place of σ_e^2).

[2] The real and imaginary noise components should be generated separately, each with variance $\sigma_w^2/2$, using `randn`.