ECE-501 **Introduction to Analog and Digital Communications** **Winter 2008**

Homework #5 Feb. 8, 2008

HOMEWORK ASSIGNMENT #5

Due Fri. Feb. 15, 2008 (in class)

Reading:

1. Ch. 3.8, 8.1, 8.2, 8.6-8.8, 8.10.

Problems:

1. As discussed in lecture, the *autocorrelation* $R_x(\tau)$ of a wide-sense stationary[1] random signal $x(t)$ specifies how similar, on average, the sample $x(t)$ is to the sample $x(t-\tau)$:

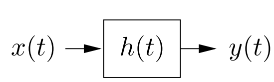
   $$R_x(\tau) = \mathrm{E}\{x(t)x^*(t-\tau)\}.$$

   Above, $\mathrm{E}\{\cdot\}$ denotes the "expectation" or statistical average, the * denotes the complex conjugate (since we allow $x(t)$ to be complex-valued), and τ is known as the "lag." Notice that $R_x(0) = \mathrm{E}\{|x(t)|^2\}$ specifies the variance of $x(t)$. The *power spectral density* $S_x(f)$ of random signal $x(t)$

   $$S_x(f) = \int_{-\infty}^{\infty} R_x(\tau)e^{-j2\pi f\tau}d\tau,$$

   measures how much average power the signal has versus frequency f. The autocorrelation and PSD form a Fourier transform pair.

   A "white" random signal has an autocorrelation of the form $N_0\delta(\tau)$, where N_0 is some positive constant and $\delta(\tau)$ is the Dirac delta, and a PSD that is constant over f with value N_0. In the lecture, we derived the following important result: when a white random signal $x(t)$ is filtered by a (non-random) filter with impulse response $h(t)$:

   the output $y(t)$ is a random signal with autocorrelation

   $$R_y(\tau) \quad = \quad N_0\int_{-\infty}^{\infty} h(t)h^*(t-\tau)dt \tag{1}$$

   and PSD

   $$S_y(f) \quad = \quad N_0|H(f)|^2. \tag{2}$$

[1] Here, "wide-sense stationary" (WSS) means that the mean and correlation are invariant to time t. We will exclusively consider WSS random signals in ECE-501.

Now say that $x(t)$ is a random signal with *generic* autocorrelation $R_x(\tau)$ and $y(t)$ is generated by filtering $x(t)$ with $h(t)$, as before.

(a) Show that

$$R_y(\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(v)h^*(u)R_x(\tau + u - v)dvdu.$$

(*Hint*: Use the same tricks used to establish (1).)

(b) Show that $S_y(f) = |H(f)|^2 S_x(f)$. (*Hint*: Use the same tricks used to establish (2).)

2. In this problem, you will experiment with the complex-baseband equivalent channel in MATLAB. Throughout the problem, assume sampling rate $T_s^{-1} = 10$ MHz.

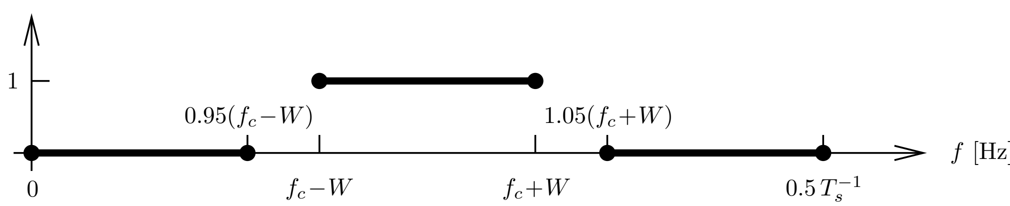
(a) Consider a 3-path channel with the following parameters:

delay	gain
0μs	1
0.4μs	-0.99
3μs	0.2

. Create a causal sampled version of the channel's impulse response $h(t)$ and evaluate its Fourier transform magnitude using `plottf`. I suggest zero-padding the impulse response to $t_{\max} = 100\mu$sec. (The amount of zero-padding affects the resolution of the frequency response plot.)

Careful: If you get an error message like "`Attempted to access ***; index must be a positive integer or logical`", it may be that you are trying to specify an integer location using a floating-point number that MATLAB does not recognize as an integer due to numerical precision issues. After verifying that the location is as you intended, simply use the `round` function to convert it to an integer.

(b) Now you will generate the *bandpass equivalent* $h_{bp}(t)$ of the wideband channel $h(t)$ above. Assume a transmission bandwidth of $2W = 1$ MHz centered at $f_c = 2.5$ MHz. To create the bandpass equivalent, filter the wideband channel with an `firls`-designed bandpass filter $B(f)$ with group delay $t_b = 100T_s$ and the following magnitude response spec:



Plot the frequency response magnitude of $h_{bp}(t)$ via `plottf`. How does the bandpass equivalent channel compare to the wideband channel?

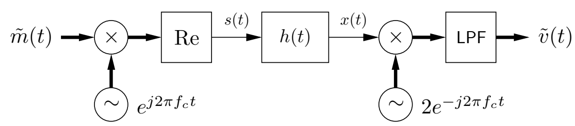
(c) Now you will generate the *complex baseband equivalent* channel $\tilde{h}(t)$. Recalling that $h_{bp}(t) = 2\,\mathrm{Re}\{\tilde{h}(t)e^{j2\pi f_c t}\}$, the easiest way of creating $\tilde{h}(t)$ is via

$$\tilde{h}(t) = \mathsf{LPF}\{h_{bp}(t)e^{-j2\pi f_c t}\}$$

where the LPF is the standard one we use in our demodulators, i.e., with passband edge at W Hz and stopband edge at $2f_c - W$ Hz. To design the LPF, I suggest using `fir2` with group delay $t_{\mathsf{LPF}} = 10T_s$. Plot the frequency response magnitude of $\tilde{h}(t)$ via `plottf`. How does the complex baseband equivalent channel compare to the passband equivalent and wideband channels?

(d) Next you will verify that the complex-baseband channel model is in fact "equivalent" to the original wideband channel model. To start, generate a real-valued random message signal $m(t)$ with length $t_{\max} = 100\mu$ sec and single-sided bandwidth W, just as you have done in previous homeworks.

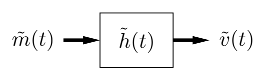
(e) AM-modulate the message at carrier frequency f_c, filter it using the wideband channel $h(t)$, and AM-demodulate the (noiseless) channel output $x(t)$. In other words, implement the block diagram below (where, for AM, $\tilde{m}(t) = m(t)$ and $\text{Re}\{\tilde{v}(t)\} = v(t)$):



Plot the recovered signal $v(t)$ via `plottf`, and superimpose the original message $m(t)$ using a dashed red line. How does $v(t)$ compare to $m(t)$?

(*Hint*: To make sure everything is working properly, test your code using a single-path unit-gain version of $h(t)$, for which $v(t)$ should be a delayed version of $m(t)$.)

(f) Now repeat the simulation of modulation/propagation/demodulation using the complex baseband equivalent channel $\tilde{h}(t)$, as illustrated in the block diagram below.



Again, plot $\text{Re}\{\tilde{v}(t)\}$ via `plottf`, and superimpose the original message $m(t)$ using a dashed red line. How does this plot compare to that of part (b)? Provide an explanation for any differences that you see.