## HOMEWORK ASSIGNMENT #4

**Due Fri. Feb. 1, 2008** (in class)

Reading:

1. Ch. 3.6–3.8.

Problems:

1. Prove that if $\mathcal{F}\{c(t)\} = C(f)$, then $\mathcal{F}\{\cos(2\pi f_c t)c(t)\} = \frac{1}{2}\left[C(f - f_c) + C(f + f_c)\right]$.

2. Complex-Baseband QAM Computer Experiment:

   Throughout, assume sampling rate $\frac{1}{Ts} = 1000$ Hz.

   (a) As in Homework #3.3(a), generate two random real-valued messages $m_{\rm I}(t)$ and $m_{\rm Q}(t)$, each with length $t_{\max} = 1$ sec and single-sided bandwidth $W = 25$ Hz, and combine them into the complex-baseband message $\tilde{m}(t) = m_{\rm I}(t) + jm_{\rm Q}(t)$. Plot the Fourier transform of the message using `plottf` with the `'f'` option. Does the complex-baseband message look as expected? Comment on the frequency-domain symmetry.

   (b) Modulate the complex message using the complex-baseband version of the QAM modulator with carrier frequency $f_c = 200$ Hz. Plot the transmitted signal $s(t)$ in the frequency domain using `plottf` with the `'f'` option. Does the transmitted signal look as expected?

   (c) Coherently demodulate the transmitted signal using a complex-baseband QAM demodulator, and split its complex-valued output $\tilde{v}(t)$ into two real-valued signals: $v_{\rm I}(t) = \mathrm{Re}\{\tilde{v}(t)\}$ and $v_{\rm Q}(t) = \mathrm{Im}\{\tilde{v}(t)\}$. You can design the demodulator's LPF as you did in Homework #3.3(b). Plot $v_{\rm I}(t)$ and $v_{\rm Q}(t)$ on separate plots (using `plottf` with the `'t'` option) superimposed with the original message signals $m_{\rm I}(t)$ and $m_{\rm Q}(t)$, as in Homework #3.3(b). How do the recovered signals compare to the original messages?

3. Passband VSB Computer Experiment:

   Throughout, assume sampling rate $\frac{1}{Ts} = 1000$ Hz.

   (a) As in Homework #3.1(a), generate a random message signal $m(t)$ with length $t_{\max} = 1$ sec and single-sided bandwidth $W = 25$ Hz. Plot the Fourier transform of the message using `plottf` with the `'f'` option. Does it look as expected?

   (b) Modulate the message using VSB with passband filtering, assuming carrier frequency $f_c = 200$ Hz. To create a passband VSB filter $c(t)$, use the routine `firvsb.m` posted on the course webpage. (Type "`help firvsb`" for instructions on usage.) I suggest a group delay of $t_c = 100$ msec and rolloff parameter $\alpha = 0.1$ (though any $\alpha \in [0, 1]$ will work). Plot the transmitted signal $s(t)$ in the frequency domain using `plottf` with the `'f'` option. Does the transmitted signal look as expected?

(c) Plot the passband-VSB filter's frequency response $|C(f)|$ using `plottf` with the `'f'` option. Then, to ascertain whether $C(f)$ satisfies the essential property

$$\frac{1}{2}\Big[C(f-f_c)+C(f+f_c)\Big]=1 \text{ for } |f|\leq W,$$

plot the Fourier transform of $\cos(2\pi f_c t)c(t)$ via `plottf.m` with the `'f'` option. (Recall the property proven in Problem 1 of this assignment.) Do the plots look as expected?

(d) Coherently demodulate the transmitted signal using an AM demodulator with a small but important modification: *the cosine waveform must be delayed by $t_c$ seconds, where $t_c$ is the group delay of the passband VSB filter.* You can design the demodulator's LPF as you did in Homework #3.1(c). Plot the recovered signal in time and frequency domains using `plottf`, then superimpose the original message on the time-domain plot as you did in Homework # 3.1(c). How does the recovered signal compare to the original message?

4. Complex-Baseband VSB Computer Experiment:
Throughout, assume sampling rate $\frac{1}{Ts}=1000$ Hz.

(a) Repeat 3(a) to generate a random message signal $m(t)$ with length $t_{\max}=1$ sec and single-sided bandwidth $W=25$ Hz.

(b) Modulate the message using VSB with complex-baseband filtering, assuming carrier frequency $f_c=200$ Hz. To create a complex baseband VSB filter $\tilde{c}(t)$, you can use the routine `firvsb.m` again with appropriate arguments. (Type "`help firvsb`" for instructions on usage.) Plot the transmitted signal in time and frequency domains using `plottf`. Does the transmitted signal look as expected?

(c) Plot the baseband-VSB filter's frequency response $|\tilde{C}(f)|$ using `plottf` with the `'f'` option. Then, to ascertain whether $\tilde{C}(f)$ satisfies the essential property

$$\frac{1}{2}\Big[\tilde{C}(f)+\tilde{C}^*(-f)\Big]=1 \text{ for } |f|\leq W,$$

plot the Fourier transform of $\text{Re}\{\tilde{c}(t)\}$ via `plottf.m` with the `'f'` option. (Why does this work?) Do the plots look as expected?

(d) Coherently demodulate the transmitted signal using the same demodulator as in 3(d). Plot the recovered signal in time and frequency domains using `plottf`, then superimpose the original message on the time-domain plot as you did in 3(d). How does the recovered signal compare to the original message?