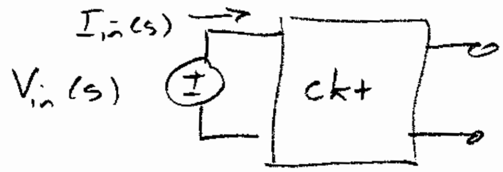
ECE 301 MIDTERM #2 SOLUTIONS AU06

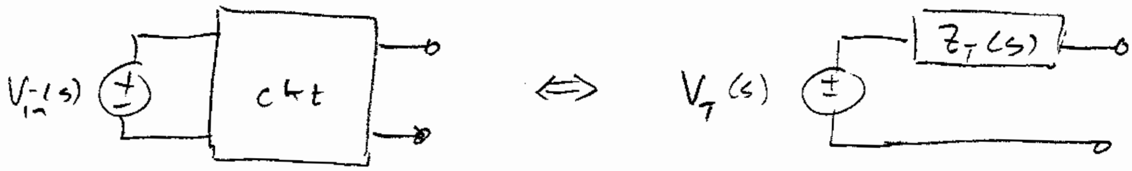
1) • Input impedance can be calculated as

$$Z_{in}(s) = \frac{V_{in}(s)}{I_{in}(s)}$$ where



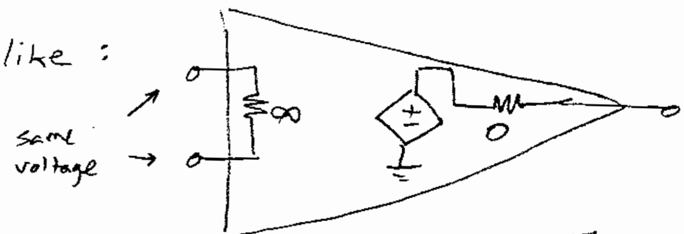
Approach: Solve for $I_{in}(s)$ in terms of general $V_{in}(s)$

• Output impedance is equivalent to the Thevenin impedance:

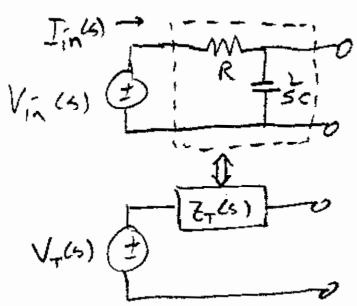


Approach: Calculate $Z_T(s)$ assuming general $V_{in}(s)$

• But remember Op-Amps behave like:



a)



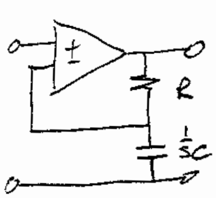
$$I_{in}(s) = \frac{V_{in}(s)}{R + \frac{1}{sC}} \Rightarrow Z_{in}(s) = \frac{V_{in}(s)}{I_{in}(s)} = \boxed{R + \frac{1}{sC}}$$

$$Z_T(s) = \frac{V_{oc}(s)}{I_{sc}(s)} = \boxed{\frac{R}{1+sRC}}$$ since

$$V_{oc}(s) = V_{in}(s)\,\frac{1/sC}{1/sC + R} = V_{in}(s)\,\frac{1}{1+sRC}$$

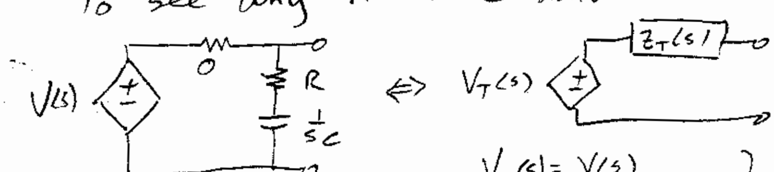
$$I_{sc}(s) = \frac{V_{in}(s)}{R}$$

b)



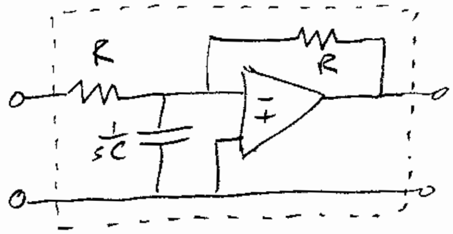
We've discussed this "non-inverting configuration" many times, which has $Z_{in}(s) = \boxed{\infty}$ and $Z_{out}(s) = \boxed{0}$

To see why R and C don't affect $Z_{out}(s)$, consider

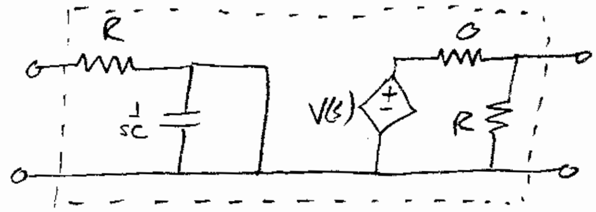


$$\left.\begin{aligned} V_{oc}(s) &= V(s) \\ I_{sc}(s) &= \frac{V(s)}{0} = \infty \end{aligned}\right\} \; Z_T(s) = \frac{V_{oc}(s)}{I_{sc}(s)} = 0 \; \checkmark$$

c)

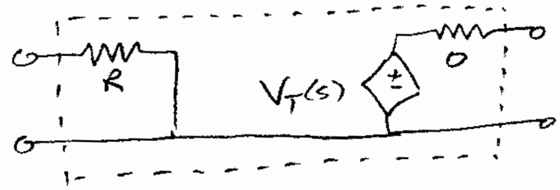


$\Longleftrightarrow$



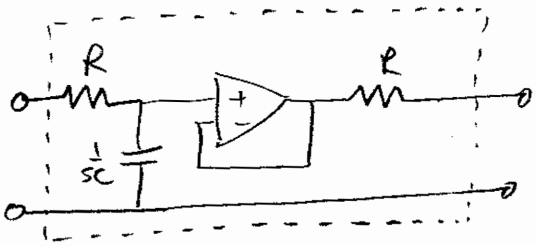
Realizing that the input terminals are at the same voltage

$\Updownarrow$

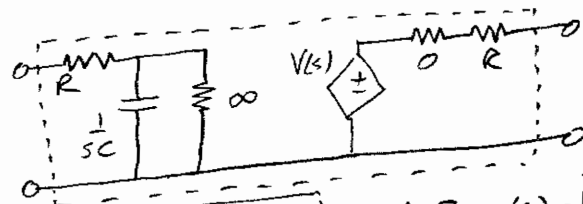


Thus $Z_{in}(s) = \boxed{R}$ and $Z_{out}(s) = \boxed{0}$

d)



$\Longleftrightarrow$



Thus $Z_{in}(s) = \boxed{R + \frac{1}{sC}}$ and $Z_{out}(s) = \boxed{R}$

2) Impulse response $e^{-2t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+2}$.

In other words $\underbrace{Y(s)}_{\frac{1}{s+2}} = T(s) \underbrace{X(s)}_{1}$, so that $T(s) = \frac{1}{s+2}$

a) The steady state response to $10\cos(5t)$ equals

$10\,|T(j5)|\cos(5t + \measuredangle T(j5))$, where

$$|T(j5)| = \left|\frac{1}{j5+2}\right| = \frac{1}{\sqrt{5^2+2^2}} = \frac{1}{\sqrt{29}} = 0.186$$

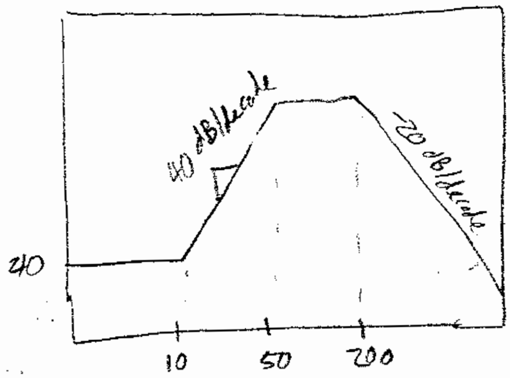
$$\measuredangle T(j5) = \measuredangle\left(\frac{1}{j5+2}\right) = -\measuredangle(j5+2) = -\tan^{-1}\left(\frac{5}{2}\right) = -68.2^\circ$$

$= \boxed{1.86\cos(5t - 68.2^\circ)}$

b) Step response: $G(s) = T(s)\,\frac{1}{s} = \frac{1}{s(s+2)} = \frac{k_1}{s} + \frac{k_2}{s+2}$

Cover-up yields $k_1 = \frac{1}{2}$ & $k_2 = -\frac{1}{2}$, so that $\boxed{g(t) = \left[\frac{1}{2} - \frac{1}{2}e^{-2t}\right]u(t)}$

3a)



So two zeros @ $\omega = 10$
two poles @ $\omega = 50$
one pole @ $\omega = 200$
DC gain = 40dB = 100

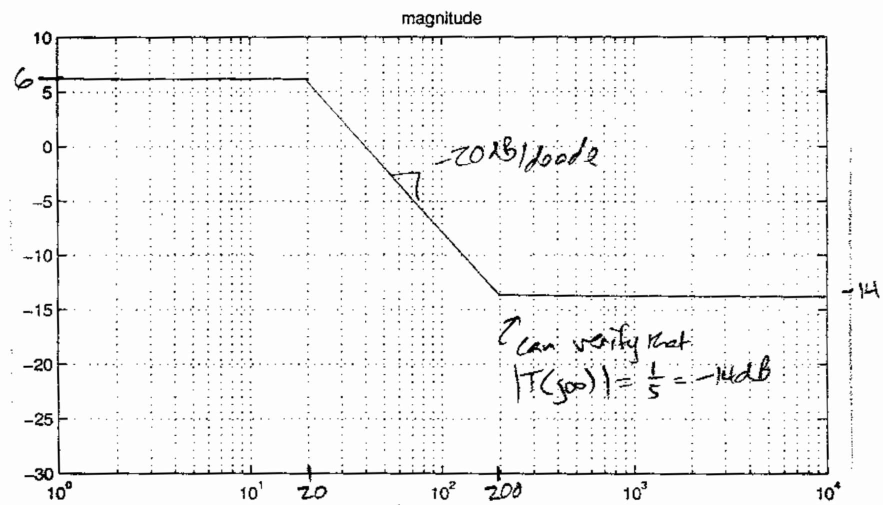
$$\Rightarrow \quad T(s) = \frac{100\left(\frac{s}{10}+1\right)^2}{\left(\frac{s}{50}+1\right)^2\left(\frac{s}{200}+1\right)}$$

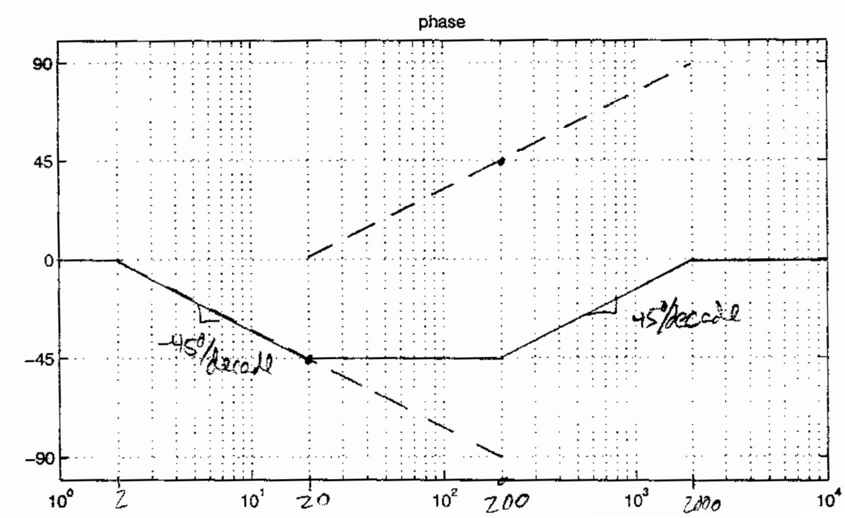
note that $\left(\frac{s}{\alpha}+1\right)$ has DC gain $=1$, thus 100 appears in numerator.
whereas $(s+\alpha)$ does not have DC gain $=1$ (unless $\alpha=1$) !

b)

$$T(s) = \frac{s+200}{5s+100} = \frac{200}{100}\frac{\left(\frac{s}{200}+1\right)}{\left(\frac{5s}{100}+1\right)} = 2\,\frac{\left(\frac{s}{200}+1\right)}{\left(\frac{s}{20}+1\right)}$$

So DC gain $= 2 = 6$dB
zero at $\omega = 200$
pole at $\omega = 20$

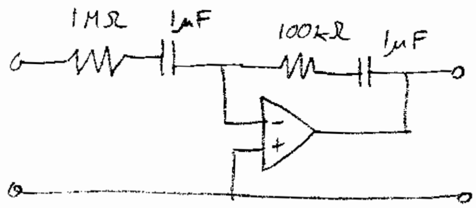




4) There were a few designs that implemented $-\frac{1}{10}\cdot\frac{s+10}{s+1}$ using a single op-amp, no inductors, and 1μF capacitors.

i) $$-\frac{1}{10}\cdot\frac{s+10}{s+1} = -\frac{\frac{s}{10}+1}{s+1} = -\frac{\frac{1}{10}+\frac{1}{s}}{1+\frac{1}{s}} = -\frac{\frac{10^6}{10}+\frac{10^6}{s}}{10^6+\frac{10^6}{s}} = -\frac{10^5+\frac{1}{10^{-6}s}}{10^6+\frac{1}{10^{-6}s}} = -\frac{Z_2(s)}{Z_1(s)}$$

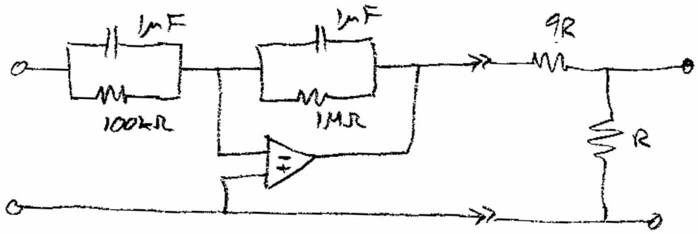
for inverting config



$10^6 = 1M$

$10^5 = 100k$

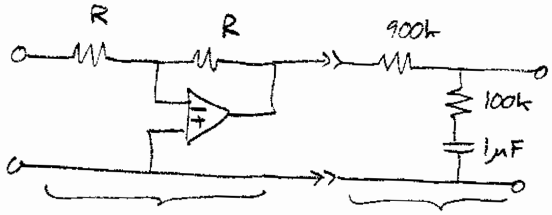
ii) $$-\frac{1}{10}\cdot\frac{s+10}{s+1} = -\frac{1}{10}\cdot\frac{(s+1)^{-1}}{(s+10)^{-1}} = -\frac{1}{10}\frac{\left[\left(\frac{1}{s}\right)^{-1}+(1)^{-1}\right]^{-1}}{\left[\left(\frac{1}{s}\right)^{-1}+\left(\frac{1}{10}\right)^{-1}\right]^{-1}} = -\frac{\left[\left(\frac{1}{10^{-6}s}\right)^{-1}+\left(\frac{1}{10^{-6}}\right)^{-1}\right]^{-1}}{\left[\left(\frac{1}{10^{-6}s}\right)^{-1}+\left(\frac{1}{10\cdot10^{-6}}\right)^{-1}\right]^{-1}}\cdot\frac{1}{10} = -\underbrace{\frac{\left[\left(\frac{1}{10^{-6}s}\right)^{-1}+\left(10^{6}\right)^{-1}\right]^{-1}}{\left[\left(\frac{1}{10^{-6}s}\right)+\left(10^{5}\right)^{-1}\right]^{-1}}}_{\text{inverting}}\cdot\underbrace{\frac{1}{10}}_{\text{voltage divider}}$$



← note: must cascade with op-amp first!

for any R

iii)

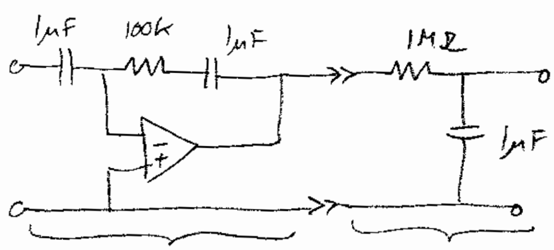


for any R

-1

$$\frac{s+10}{10s+10} = \frac{1+\frac{10}{s}}{10+\frac{10}{s}} = \frac{1+\frac{10}{s}}{9+\left(1+\frac{10}{s}\right)} = \frac{10^5+\frac{10^6}{s}}{9\times10^5+\left(10^5+\frac{10^6}{s}\right)}$$

iv)



$$-\frac{s+10}{10} = -\frac{0.1+s^{-1}}{s^{-1}} \qquad \frac{1}{s+1} = \frac{s^{-1}}{1+s^{-1}} = \frac{10^6 s^{-1}}{10^6+10^6 s^{-1}}$$

Note: the output impedances of the last two designs are extremely high, hence these are "unfriendly" circuits. The first and simplest design is the "friendliest"!