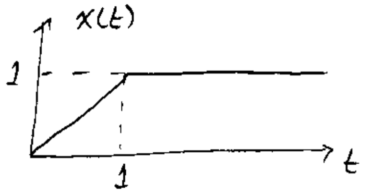
EE 301 MIDTERM 1 SOLUTIONS

1 a) $x(t) = \delta(t) + 5t\,u(t) + e^{-10t}\cos(2t)\,u(t)$

$$X(s) = 1 + \frac{5}{s^2} + \frac{s+10}{(s+10)^2+2^2} = 1 + \frac{5}{s^2} + \frac{s+10}{s+20s+104}$$

b) 

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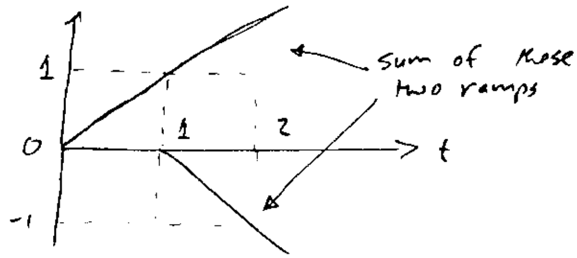


$$x(t) = r(t) - r(t-1)$$
$$= t\,u(t) - (t-1)\,u(t-1)$$
$$X(s) = \frac{1}{s^2} - \frac{e^{-s}}{s^2} = \frac{1-e^{-s}}{s^2}$$

c) $$X(s) = \frac{s+2}{s^2+s} = \frac{s+2}{s(s+1)} = \frac{k_1}{s} + \frac{k_2}{s+1}$$

$$\left.\begin{aligned} k_1 &= \left.\frac{s+2}{s+1}\right|_{s=0} = 2 \\ k_2 &= \left.\frac{s+2}{s}\right|_{s=-1} = -1 \end{aligned}\right\} \quad \begin{aligned} X(s) &= \frac{2}{s} - \frac{1}{s+1} \\ x(t) &= 2u(t) - e^{-t}u(t) \end{aligned}$$

d) $$X(s) = \frac{s+1}{s^2+2s+5}$$

method #1: $$X(s) = \frac{s+1}{(s+1)^2+2^2} = e^{-t}\cos(2t)\,u(t)$$

method #2: $$X(s) = \frac{s+1}{(s-(-1-j2))(s-(-1+j2))} = \frac{k}{s+1+j2} + \frac{k^*}{s+1-j2}$$

$$k = \left.\frac{s+1}{s+1-j2}\right|_{s=-1-j2} = \frac{-j2}{-j4} = \frac{1}{2} \quad \text{so} \quad \begin{cases} |k| = \frac{1}{2} \\ \angle k = 0° \\ \alpha = 1 \\ \beta = 2 \end{cases}$$

$$x(t) = 2|k|\,e^{-\alpha t}\cos(\beta t + \angle k)\,u(t)$$
$$= e^{-t}\cos(2t)\,u(t)$$

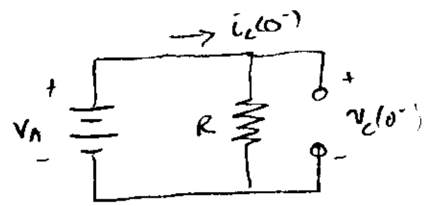
2) Using the property that, at steady state,

inductors ⟺ shorts

capacitors ⟺ open circuits

we can rewrite the block diagrams just before $t=0$ as follows...

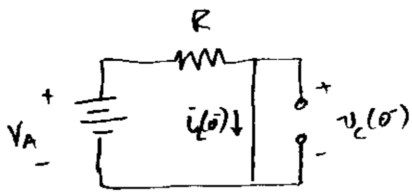
a)



$v_C(0^-) = V_A$

$i_L(0^-) = \frac{V_A}{R}$

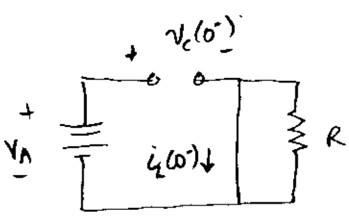
b)



$v_C(0^-) = 0$

$i_L(0^-) = \frac{V_A}{R}$

c)

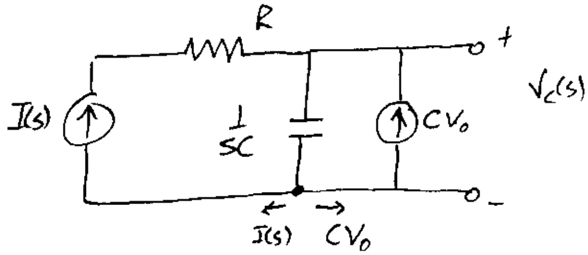


$v_C(0^-) = V_A$

$i_L(0^-) = 0$

3) method #1:

a)

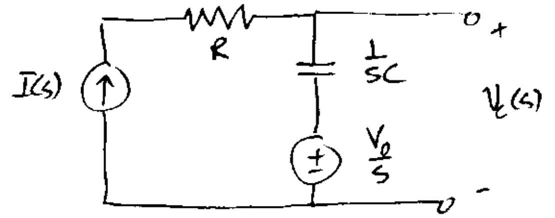


$I(s) = \frac{1}{s+10}$

b) $V_c(s) = \frac{1}{sC}\left(I(s) + CV_0\right) = \frac{1}{sC(s+10)} + \frac{V_0}{s} = \frac{1/C}{s(s+10)} + \frac{V_0}{s}$

method #2:

a)



$I(s) = \frac{1}{s+10}$

b) $V_c(s) = \frac{V_0}{s} + I(s)\frac{1}{sC} = \frac{V_0}{s} + \frac{1}{(s+10)sC} = \frac{1/C}{s(s+10)} + \frac{V_0}{s}$

c) To find the forced/natural poles, consider the zero-state transfer function from forced input to output:

$$V_c(s)\Big|_{\text{zero-state}} = \frac{1}{sC}\cdot I(s) = \frac{1/C}{s}\cdot\frac{1}{s+10}$$

forced pole at $s=-10$

natural pole at $s=0$

d) $V_c(s) = \frac{1/C}{s(s+10)} + \frac{V_0}{s}$

$$\frac{\frac{1}{C}}{s(s+10)} = \frac{k_1}{s} + \frac{k_2}{s+10} \qquad \begin{cases} k_1 = \left.\dfrac{1/C}{s+10}\right|_{s=0} = \dfrac{1}{10C} \\ k_2 = \left.\dfrac{1/C}{s}\right|_{s=-10} = -\dfrac{1}{10C} \end{cases}$$

$$V_c(s) = \frac{1}{10C}\left(\frac{1}{s}\right) - \frac{1}{10C}\left(\frac{1}{s+10}\right) + \frac{V_0}{s}$$

$$v_c(t) = \left(\frac{1}{10C} + V_0\right)u(t) - \frac{1}{10C}e^{-10t}u(t)$$

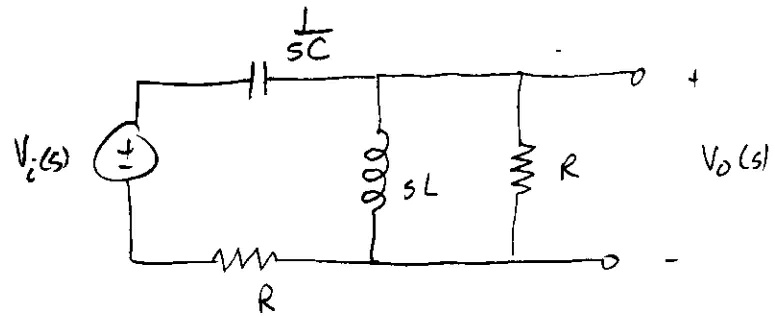
e) Components proportional to the initial condition are zero-input:

$$v_c(t)\Big|_{\text{zero-input}} = V_0\, u(t)$$

The rest are zero-state:

$$v_c(t)\Big|_{\text{zero-state}} = \frac{1}{10C}u(t) - \frac{1}{10C}e^{-10t}u(t)$$

4) While it is possible to do a mesh-current or node-voltage analysis, it is much easier to apply voltage division :



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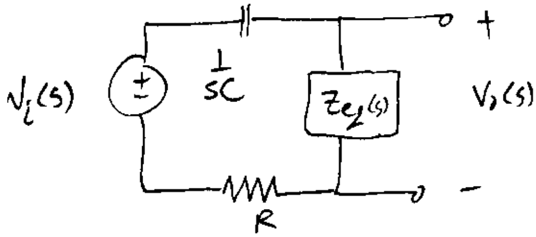

$$Z_{eq}(s) = \left(\frac{1}{sL} + \frac{1}{R}\right)^{-1} = \frac{sLR}{sL+R}$$

$$V_o(s) = \frac{Z_{eq}(s)}{\frac{1}{sC} + R + Z_{eq}(s)} = \frac{\frac{sLR}{sL+R}}{\left(\frac{1}{sC} + R\right) + \frac{sLR}{sL+R}}$$

$$= \frac{sLR}{\left(\frac{1}{sC} + R\right)(sL+R) + sLR} = \frac{sLR}{\frac{L}{C} + \frac{R}{sC} + sRL + R^2 + sRL}$$

$$= \frac{s^2LR}{2s^2RL + s\left(\frac{L}{C} + R^2\right) + \frac{R}{C}} = \frac{s^2/2}{s^2 + s\left(\frac{R^2 + L/C}{2LR}\right) + \frac{1}{2LC}}$$