31, ~~10.53~~, ~~10.36~~, ~~10.39~~

ECE 301 B3

40/40

10/18/06 [redacted]

10-31)

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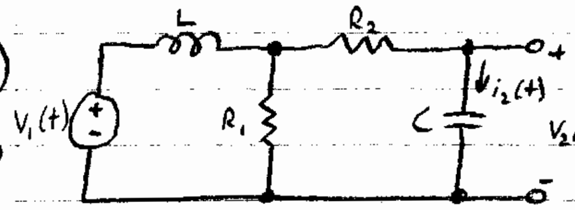
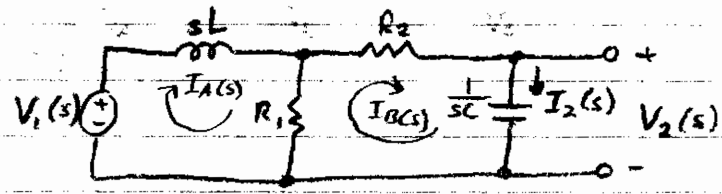


⇒



There is no initial energy

A) Transform circuit into s-domain and find mesh-current eq.

$$sLI_A(s) + R_1(I_A(s) - I_B(s)) - V_1(s) = 0$$

$$R_2 I_B(s) + \frac{1}{Cs} I_B(s) + R_1(I_B(s) - I_A(s)) = 0 \qquad I_B(s) = I_2(s)$$

$$\boxed{\begin{aligned} (sL+R_1)I_A(s) - R_1 I_2(s) &= V_1(s) \\ -R_1 I_A(s) + (R_2 + 1/Cs + R_1) I_B(s) &= 0 \end{aligned}}$$

B)
$$\begin{bmatrix} sL+R_1 & -R_1 \\ -R_1 & R_2 + 1/Cs + R_1 \end{bmatrix} \begin{bmatrix} I_A(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} V_1(s) \\ 0 \end{bmatrix}$$

$$\Delta_2(s) = \begin{bmatrix} sL+R_1 & V_1(s) \\ -R_1 & 0 \end{bmatrix} = R_1 V_1(s)$$

$$\Delta(s) = \begin{bmatrix} sL+R_1 & -R_1 \\ -R_1 & R_2 + 1/Cs + R_1 \end{bmatrix} = R_2Ls + R_1R_2 + L/C + R_1/Cs + R_1Ls + \cancel{R_1^2} - \cancel{R_1^2}$$
$$= \cancel{R_2LCs^2 + R_1R_2Cs + Ls + R_1 + R_1LCs^2}$$
$$= \frac{(R_1+R_2)LCs^2 + (R_1R_2C + L)s + R_1}{Cs}$$

$$I_2(s) = \frac{\Delta_2(s)}{\Delta(s)} = \boxed{\frac{R_1 C s V_1(s)}{(R_1+R_2)LCs^2 + (R_1R_2C+L)s + R_1}}$$

c) Find $i_2(t)$ for $v_1(t) = 100u(t)$ V, $R_1 = 1k\Omega$, $R_2 = 2k\Omega$, $L = 4H$, $C = 500nF$

$$I_2(s) = \frac{1/20}{\frac{3}{500}s^2 + 5s + 1000} = \frac{25}{3s^2 + 2500s + 500000} = \frac{25}{(s+500)(3s+1000)}$$

$P_1 = -500$ $\quad K_1 = \frac{25}{3(-500)+1000} = -1/20$ $\qquad P_2 = -1000/3$ $\quad K_2 = \frac{25}{-\frac{1000}{3} + 500} = 3/20$

$$I_2(s) = \frac{-1/20}{s+500} + \frac{1/20}{s+1000/3}$$

$$i_2(t) = \left[-\tfrac{1}{20} e^{-500t} + \tfrac{1}{20} e^{-1000t/3}\right] u(t) \text{ A}$$

$$\boxed{i_2(t) = \left[-50 e^{-500t} + 50 e^{-1000t/3}\right] u(t) \text{ mA}}$$

10-33)

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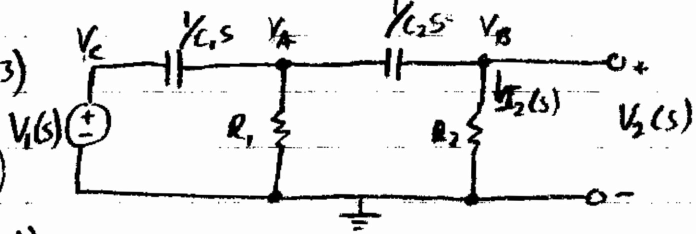


$V_B = V_2(s)$

A)

$$\boxed{\begin{aligned} &\text{Node A: } (C_1 s + 1/R_1 + C_2 s) V_A - C_2 s V_2(s) = C_1 s V_1(s) \\ &\text{Node B: } (C_2 s + 1/R_2) V_2(s) - C_2 s V_A = 0 \end{aligned}}$$

B)

$$\begin{bmatrix} C_1 s + 1/R_1 + C_2 s & -C_2 s \\ -C_2 s & C_2 s + 1/R_2 \end{bmatrix} \begin{bmatrix} V_A \\ V_2(s) \end{bmatrix} = \begin{bmatrix} C_1 s V_1(s) \\ 0 \end{bmatrix}$$

$$\Delta_2(s) = \begin{bmatrix} C_1 s + 1/R_1 + C_2 s & C_1 s V_1(s) \\ -C_2 s & 0 \end{bmatrix} = C_1 C_2 s^2 V_1(s)$$

$$\Delta(s) = \begin{bmatrix} C_1 s + 1/R_1 + C_2 s & -C_2 s \\ -C_2 s & C_2 s + 1/R_2 \end{bmatrix} = C_1 C_2 s^2 + \frac{C_2 s}{R_1} + \frac{C_2 s}{R_2} + \cancel{(C_2 s)^2} + \frac{1}{R_1 R_2} + \frac{C_1 s}{R_2}$$

$$= R_1 R_2 C_1 C_2 s^2 + R_2 C_2 s + R_1 C_2 s + 1 + R_1 C_1 s$$

$$= \frac{R_1 R_2 C_1 C_2 s^2 + (R_2 C_2 + R_1 C_2 + R_1 C_1) s + 1}{R_1 R_2}$$

$$V_2(s) = \frac{\Delta_2(s)}{\Delta(s)} = \boxed{\frac{R_1 R_2 C_1 C_2 s^2 V_1(s)}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_2 C_2 + R_1 C_2) s + 1}}$$

c)

$$V_2(s) = \frac{.003 s}{.0001 s^2 + .025 s + 1} = \frac{30 s}{s^2 + 250 s + 10000} = \frac{30 s}{(s+50)(s+200)}$$

$p_1 = -50 \qquad K_1 = \frac{30(-50)}{-50+200} = -10$

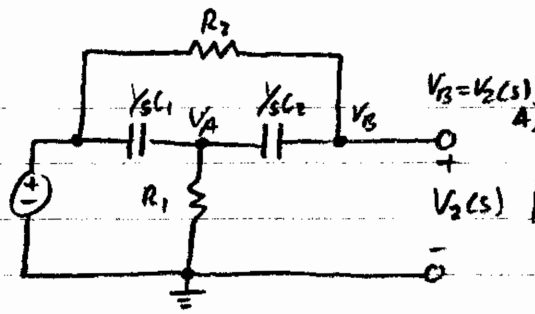
$p_2 = -200 \qquad K_2 = \frac{30(-200)}{-200+50} = 40$

$$V_2(s) = \frac{40}{s+200} - \frac{10}{s+50}$$

$$\boxed{v_2(t) = \left[40 e^{-200t} - 10 e^{-50t}\right] u(t) \text{ V}}$$

10-36)

10



A) Node A: $\left(C_1 s + \frac{1}{R_1} + C_2 s\right) V_A - C_2 s V_2(s) = C_1 s V_1(s)$

Node B: $\left(1/R_2 + C_2 s\right) V_2(s) - C_2 s V_A = \frac{V_1(s)}{R_2}$

B) $$\begin{bmatrix} C_1 s + 1/R_1 + C_2 s & -C_2 s \\ -C_2 s & 1/R_2 + C_2 s \end{bmatrix} \begin{bmatrix} V_A \\ V_2(s) \end{bmatrix} = \begin{bmatrix} C_1 s V_1(s) \\ V_1(s)/R_2 \end{bmatrix}$$

$$\Delta_2(s) = \begin{vmatrix} C_1 s + 1/R_1 + C_2 s & C_1 s V_1(s) \\ -C_2 s & V_1(s)/R_2 \end{vmatrix} = \frac{C_1 s V_1(s)}{R_2} + \frac{V_1(s)}{R_1 R_2} + \frac{C_2 s V_1(s)}{R_2} + C_1 C_2 s^2 V_1(s)$$

$$= \frac{R_1 C_1 s V_1(s) + V_1(s) + R_1 C_2 s V_1(s) + R_1 R_2 C_1 C_2 s^2 V_1(s)}{R_1 R_2}$$

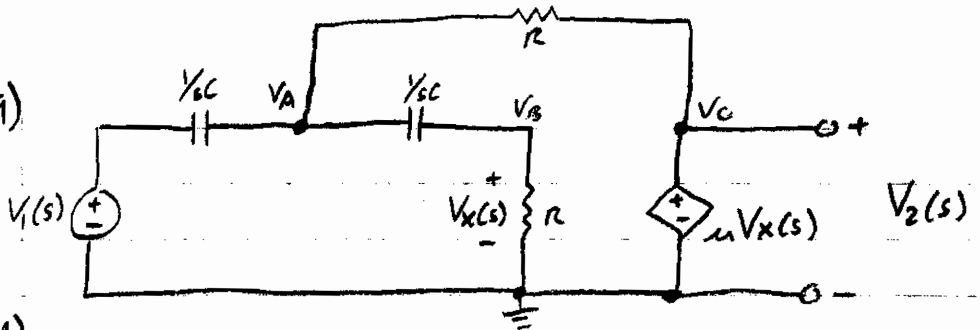
$$\Delta(s) = \begin{vmatrix} C_1 s + 1/R_1 + C_2 s & -C_2 s \\ -C_2 s & 1/R_2 + C_2 s \end{vmatrix} = \frac{C_1 s}{R_2} + \frac{1}{R_1 R_2} + \frac{C_2 s}{R_2} + C_1 C_2 s^2 + \frac{C_2 s}{R_1} + \cancel{C_2^2 s^2 - C_2^2 s^2}$$

$$= \frac{R_1 C_1 s + 1 + R_1 C_2 s + R_1 R_2 C_1 C_2 s^2 + R_2 C_2 s}{R_1 R_2}$$

$$V_2(s) = \frac{\Delta_2(s)}{\Delta(s)} = \frac{R_1 C_1 s V_1(s) + V_1(s) + R_1 C_2 s V_1(s) + R_1 R_2 C_1 C_2 s^2 V_1(s)}{R_1 C_1 s + 1 + R_1 C_2 s + R_1 R_2 C_1 C_2 s^2 + R_2 C_2 s}$$

$$\boxed{V_2(s) = \left[\frac{R_1 R_2 C_1 C_2 s^2 + (C_1 + C_2) R_1 s + 1}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_1 C_2 + R_2 C_2) s + 1}\right] V_1(s)}$$

10-39)



$V_C = \mu V_X(s) = \mu V_B$

10

A)

Node A: $\left(Cs + \frac{1}{R} + Cs\right)V_A - CsV_B - \frac{\mu V_B}{R} = CsV_1(s)$

Node B: $\left(Cs + \frac{1}{R}\right)V_B - CsV_A = 0$

$$\Delta(s) = \begin{bmatrix} 2Cs + \frac{1}{R} & -Cs - \frac{\mu}{R} \\ -Cs & Cs + \frac{1}{R} \end{bmatrix} = 2C^2s^2 + \frac{Cs}{R} + \frac{2Cs}{R} + \frac{1}{R^2} - C^2s^2 - \frac{\mu Cs}{R}$$

$$= \frac{2R^2C^2s^2 + RCs + 2RCs + 1 - R^2C^2s^2 - \mu RCs}{R^2}$$

$$= \frac{R^2C^2s^2 + 3RCs - \mu RCs + 1}{R^2}$$

$$\boxed{\Delta(s) = \frac{(RCs)^2 + (3-\mu)RCs + 1}{R^2}}$$

B) Select R, C, μ so that $\omega_0 = 5K$ rad/s and $\zeta = 1/\sqrt{2}$

$$\frac{R^2C^2s^2}{R^2} + \frac{(3-\mu)RCs}{R^2} + \frac{1}{R^2} = C^2s^2 + \frac{(3-\mu)Cs}{R} + \frac{1}{R^2}$$

$$s^2 = \frac{(3-\mu)s}{RC} + \left(\frac{1}{RC}\right)^2$$

$s^2 + 2\zeta\omega_0 s + \omega_0^2$

$$\frac{1}{RC} = \omega_0 = 5000 \qquad RC = 1/5000 = .0002$$

$$\frac{3-\mu}{RC} = 2\zeta\omega_0 = 2(5000)(1/\sqrt{2}) = 5000\sqrt{2}$$

$$3 - \mu = 5000\sqrt{2}(.0002)$$

$$\boxed{\begin{aligned} \mu &= 1.586 \\ RC &= .0002 \end{aligned}}$$