1-1, 7-15, 7-10, 7-10, 7-..

[redacted]

ECE 301 B1

10/4/06 [redacted]

9-9) Find the Laplace trasform of the following waveform. Locate the poles and zeros of F(s).

a) $f_1(t) = \delta(t) - (625 + e^{-50})u(t) \Rightarrow F_1(s) = 1 - \frac{625}{(s+50)^2} = \frac{(s+50)^2 - 625}{(s+50)^2}$

[redacted] $F_1(s) = \frac{s^2 + 100s + 2500 - 625}{(s+50)^2} = \boxed{\frac{(s+25)(s+75)}{(s+50)^2}}$

Poles: $-50, -50$
Zeros: $-25, -75$

b) $f_2(t) = [5 + e^{-20t} - 6\cos(10t) + 2\sin(10t)]\,u(t)$

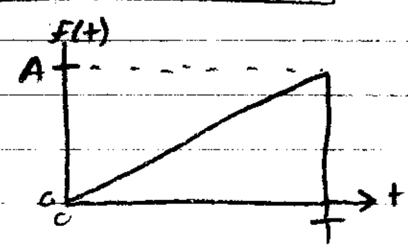
$$F_2(s) = \frac{5}{5} + \frac{1}{s+20} - \frac{6s}{s^2+10^2} + \frac{20}{s^2+10^2}$$

$$= \frac{5(s+20)(s^2+10^2) + s(s^2+10^2) - 6s(s)(s+20) + 20s(s+20)}{s(s+20)(s^2+10^2)}$$

$$= \frac{1000s + 10000}{s(s+20)(s^2+10^2)} = \boxed{\frac{1000(s+10)}{s(s+20)(s^2+10^2)}}$$

Poles: $0, -20, \pm 10j$
Zeros: -10 [redacted] [redacted]

9-13) a) Write an expression for waveform f(t) using step and ramp functions



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$$\boxed{f(t) = (A/T)\,t\,u(t) - (A/T)(t-T)u(t-T) - A\,u(t-T)}$$

b) Use the time-domain property to find the Laplace transform of the waveform f(t) found in part (a).

$$\boxed{F(s) = (A/T)\,1/s^2 - (A/T)e^{-Ts}(1/s^2) - A e^{-Ts}(1/s)}$$

c) Verify the Laplace transform found in part (b) by applying [redacted]n of the Laplace transformation in Eq. (9-2) to the waveform $f(t)$ found in part (a).

$$\text{Eq. (9-2)}: \quad F(s) = \int_{0^-}^{\infty} f(t)\, e^{-st}\, dt$$

$$F(s) = \int_0^T (A/T)\, t\, u(t)\, e^{-st}\, dt = (A/T)\left(-t/s - 1/s^2\right) e^{-st} \Big|_0^T$$

$$= (A/T)\left[\left(-T/s - 1/s^2\right) e^{-sT} + 1/s^2\right]$$

$$\boxed{F(s) = (A/T)\, 1/s^2 - (A/T)\, e^{-sT}(1/s^2) - A\, e^{-sT}(1/s)}$$

9-16) Find the inverse Laplace transforms of the following functions:

a) $$F_1(s) = \frac{s+20}{s(s+10)} = \frac{K_1}{s} + \frac{K_2}{s+10}$$

$p_1 = 0$

$$K_1 = \left.\frac{s+20}{s+10}\right|_0 = \frac{0+20}{0+10} = 2$$

$p_2 = -10$

$$K_2 = \left.\frac{s+20}{s}\right|_{-10} = \frac{-10+20}{-10} = -1$$

$$F_1(s) = \frac{2}{s} - \frac{1}{s+10} \Rightarrow \boxed{f_1(s) = \left[2 - e^{-10t}\right] u(t)}$$

b) $F_2(s) = \dfrac{20(s-20)}{(s+10)^2+400} = \dfrac{20(s-20)}{(s+10-j20)(s+10+j20)} = \dfrac{K_1}{s+10-j20} + \dfrac{K_1^*}{s+10+j20}$

$F_2(s)(s+10-j20) = \dfrac{20(s-20)}{s+10+j20}\Bigg|_{s=-10+j20} = \dfrac{20(-10+j20-20)}{\cancel{-10}+j20+\cancel{10}+j20}$

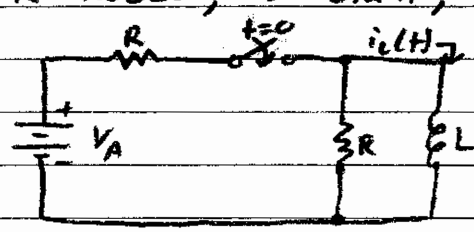
$K_1 = \dfrac{-200+j400-400}{j40} = \dfrac{-600+j400}{j40} = \dfrac{-15+j10}{j} \cdot \dfrac{j}{j} = \dfrac{-j15-10}{-1} = \boxed{10+j15}$

$|K_1| = |10+j15| = \sqrt{10^2+15^2} = 5\sqrt{13}$ $\quad \angle K_1 = \tan^{-1}(15/10) = 56.31°$

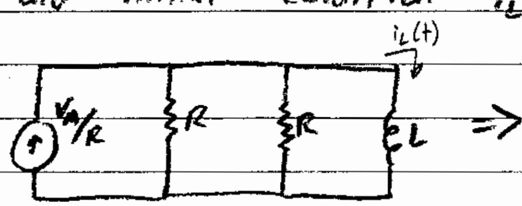
$$\boxed{f_2(t) = \left[10\sqrt{13}\, e^{-10t} \cos(20t+56.31°)\right] u(t)}$$

9-33) The switch in this circuit has been open for a long time and is closed at $t=0$. The circuit parameters

10 are $R = 200\Omega$, $L = 0.2$ H, and $V_A = 10$V

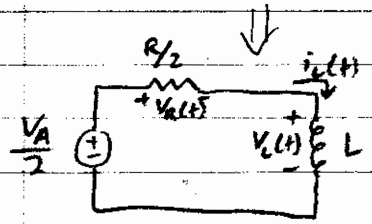


a) Find the differential eq. for the inductor current $i_L(t)$ and initial condition $i_L(0)$.



$\Rightarrow$ $\dfrac{1}{R_{eq}} = \dfrac{1}{R} + \dfrac{1}{R} = \dfrac{2}{R}$ $\quad R_{eq} = \dfrac{R}{2}$

$\Downarrow$



$V_A/2 = V_R(t) + V_L(t) \Rightarrow V_A/2 = i_L(t)(R/2) + L\dfrac{di_L(t)}{dt}$

$V_R(t) = i_L(t)(R/2)$ $\qquad 5u(t) = i_L(t)\,100 + .2\dfrac{di_L(t)}{dt}$

$V_L(t) = L\dfrac{di_L(t)}{dt}$

$$\boxed{.002\frac{di_L(t)}{dt} + i_L(t) = .05u(t) \qquad i_L(0) = 0}$$

b) $F_2(s) = \frac{s^2+10s+10}{s(s+10)}$

$$\begin{array}{r} 1 + \frac{10}{s^2+10s} \\ s^2+10s \,\overline{)\, s^2+10s+10} \\ \underline{-s^2-10s-0} \\ 10 \end{array}$$

$$\frac{10}{s^2+10s} = \frac{10}{s(s+10)} = \frac{K_1}{s} + \frac{K_2}{s+10}$$

$p_1 = 0$

$$K_1 = \left.\frac{10}{s+10}\right|_0 = \frac{10}{0+10} = 1$$

$p_2 = -10$

$$K_2 = \left.\frac{10}{s}\right|_{-10} = \frac{10}{-10} = -1$$

$$F_2(s) = 1 + \frac{1}{s} - \frac{1}{s+10} \Rightarrow \boxed{f_2(t) = \delta(t) + \left[1 - e^{-10t}\right]u(t)}$$

9-18) Find the inverse Laplace transforms of the following functions:

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a) $F_1(s) = \frac{20(s+20)}{(s+10)^2+400} = \frac{20(s+20)}{s^2+20s+500}$ $\qquad \frac{-20 \pm \sqrt{400-2000}}{2} = -10 \pm j20$

$$= \frac{20(s+20)}{(s+10-j20)(s+10+j20)} = \frac{K_1}{s+10-j20} + \frac{K_1^*}{s+10+j20}$$

$$F_1(s)(s+10-j20) = \left.\frac{20(s+20)}{s+10+j20}\right|_{s=-10+j20} = \frac{20(-10+j20+20)}{\cancel{-10}+j20\cancel{+10}+j20}$$

$$K_1 = \frac{-200+j400+400}{j40} = \frac{200+j400}{j40} = \frac{5+j10}{j} \cdot \frac{j}{j} = \frac{5j-10}{-1} = \boxed{10-j5}$$

$$|K_1| = |10-j5| = \sqrt{10^2+5^2} = 11.18 \qquad \angle K_1 = \tan^{-1}(-5/10) = -26.57^\circ$$

$$f(t) = \left[2|K|e^{-\alpha t}\cos(\beta t + \angle K)\right]u(t)$$

$$\boxed{f_1(t) = \left[22.36\, e^{-10t}\cos(20t - 26.57^\circ)\right]u(t)}$$

b) Solve for $i_L(t)$ using the Laplace transformation

$$5u(t) = i_L(t)100 + .2\frac{di_L(t)}{dt}$$

$$5/s = 100 I_L(s) + .2\left[s I_L(s) - I_o\right]$$

$$5/s = 100 I_L(s) + .2 s I_L(s) - .2 I_o$$

$$5/s + .2 I_o = I_L(s)(100 + .2s) \qquad I_L(s) = \frac{\frac{5 + .2 I_o s}{s}}{100 + .2s} = \frac{5 + .2 I_o s}{s(100 + .2s)}$$

$$I_o = 0 \qquad I_L(s) = \frac{25 + .05(0)}{s(500 + s)} = \frac{K_1}{s} + \frac{K_2}{500+s} \qquad = \frac{25 + I_o s}{s(500+s)}$$

$p_1 = 0$

$$K_1 = \left.\frac{25}{500+s}\right|_0 = \frac{25}{500} = .05$$

$p_2 = -500$

$$K_2 = \left.\frac{25}{s}\right|_{-500} = \frac{25}{-500} = -.05$$

$$I_L(s) = \frac{.05}{s} + \frac{.05}{s+500} \implies i_L(t) = .05u(t) - .05e^{-500t}u(t)$$

$$= .05\left[1 - e^{-500t}\right]u(t) \text{ A}$$

$$\boxed{i_L(t) = 50\left[1 - e^{-500t}\right]u(t) \text{ mA}}$$