Problem 9-17: Find the Inverse Laplace transforms

(a) $F_1(s) = \frac{2(s+5)}{s(s+10)} = \frac{k_1}{s} + \frac{k_2}{s+10}$ $\quad p_1 = 0\ ;\ p_2 = -10$

$$k_1 \longrightarrow sF_1(s)\Big|_0 = \frac{2(s+5)}{(s+10)}\Big|_0 = \frac{10}{10} = 1$$

$$k_2 \longrightarrow (s+10)F_1(s)\Big|_{-10} = \frac{2(s+5)}{s}\Big|_{-10} = \frac{2(-5)}{-10} = 1$$

$$F_1(s) = \frac{1}{s} + \frac{1}{s+10}$$

$$\boxed{f(t) = \left(1 + e^{-10t}\right)u(t)}$$

(b) $F_2(s) = \frac{s^2}{(s+5)(s+10)}$ $\quad n = m = 2 \longrightarrow$ long division

$$\begin{array}{r} 1 \\ s^2+15s+50 \overline{)\ s^2} \\ \underline{s^2+15s+50} \\ -15s-50 \end{array}$$

$$F_2(s) = 1 - \frac{15s+50}{(s+5)(s+10)} \qquad p_1 = -5\ ;\ p_2 = -10$$

$$k_1 \Rightarrow (s+5)F_2(s)\Big|_{-5} = \frac{15s+50}{s+10}\Big|_{-5} = \frac{-75+50}{5} = -5$$

$$k_2 \Rightarrow (s+10)F_2(s)\Big|_{-10} = \frac{15s+50}{s+5}\Big|_{-10} = \frac{-150+50}{-5} = 20$$

$$F_2(s) = 1 - \left[\frac{-5}{s+5} + \frac{20}{s+10}\right]$$

$$F_2(s) = 1 + \frac{5}{s+5} - \frac{20}{s+10}$$

$$\boxed{f(t) = \delta(t) + \left(5e^{-5t} - 20\,e^{-10t}\right)u(t)}$$