Homework: 9-2

[redacted]

- Find the Laplace Transform of $f(t) = A\left[(2-\alpha t)e^{-\alpha t}\right]u(t)$
- Locate the poles and the zeroes

$$f(t) = A\left[2e^{-\alpha t} - \alpha t e^{-\alpha t}\right]u(t)$$
$$= \left[2Ae^{-\alpha t} - A\alpha t e^{-\alpha t}\right]u(t)$$

- By Linearity: $\mathcal{L}\{f(t)\} = \mathcal{L}\{2Ae^{-\alpha t}u(t)\} - \mathcal{L}\{A\alpha t e^{-\alpha t}u(t)\}$

$$= 2A\,\mathcal{L}\{e^{-\alpha t}u(t)\} - A\alpha\,\mathcal{L}\{te^{-\alpha t}u(t)\}$$

$$= 2A \cdot \frac{1}{s+\alpha} - A\alpha \cdot \frac{1}{(s+\alpha)^2}$$

$$= \frac{2A}{s+\alpha} \cdot \frac{s+\alpha}{s+\alpha} - \frac{A\alpha}{(s+\alpha)^2}$$

$$= \frac{2As + 2A\alpha - A\alpha}{(s+\alpha)^2}$$

$$\boxed{F(s) = \frac{2As + A\alpha}{(s+\alpha)^2}}$$

- now for finding the poles, we put F(s) in scale form: $F(s) = K\frac{(s-z_1)\ldots(s-z_n}{(s-p_1)\cdots(s-p}$

$$F(s) = 2A\frac{(s+\frac{1}{2}\alpha)}{(s+\alpha)(s+\alpha)}$$

$K = 2A$

$Z_1 = -\frac{1}{2}\alpha$ ← zero

$P_1 = -\alpha$ ← pole

$P_2 = -\alpha$ ← pole