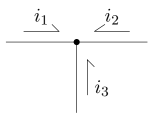
Ohm's law:

$$v = iR, \quad i = vG$$

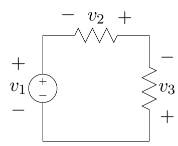
Kirchoff's law's:

KCL



$$i_1 + i_2 + i_3 = 0$$

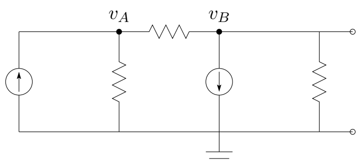
KVL



$$v_1 + v_2 + v_3 = 0$$

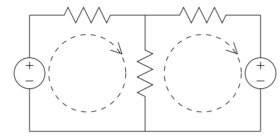
Node voltage analysis:

1. KCL at each node
2. substitute $i = vG$ at each resistor
3. solve system of equations for node voltages



Mesh current analysis:

1. KVL around each loop
2. substitute $v = iR$ at each resistor
3. solve system of equations for mesh currents



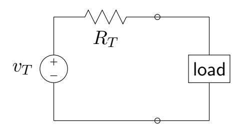
Superposition:

1. solve circuit assuming only one voltage/current source is active
2. add voltages/currents from each individual solution

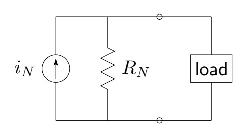
. . . a consequence of our focus on *linear* circuits.

Thevenin/Norton equivalents:

Possible to reduce linear sources to either. . .



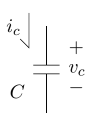
v_T = open circuit voltage



i_N = short circuit current

$$R_T = R_N = \frac{v_T}{i_N}$$

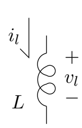
Capacitance:



$$i_c(t) = C\frac{dv_c(t)}{dt}$$

1. constant voltage across ⇔ no current thru
2. no sudden voltage changes (would require infinite current!)
3. can store as well as release energy

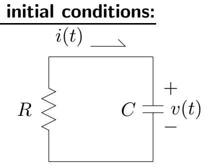
Inductance:



$$v_l(t) = L\frac{di_l(t)}{dt}$$

1. fixed current thru ⇔ no voltage across
2. no sudden current changes (would require infinite voltage!)
3. can store as well as release energy

RC response to initial conditions:



$$v(0) = V_0$$

$$v(t)\big|_{t\geq 0} = \;???$$

KVL yields (for $t \geq 0$)

$$\begin{aligned} 0 &= v(t) + i(t)R \\ &= v(t) + RC\frac{dv(t)}{dt} \end{aligned}$$

. . . a differential equation

Trying $v(t) = Ke^{st}, t \geq 0$

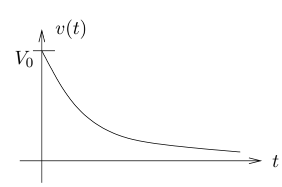
$$\begin{aligned} 0 &= Ke^{st} + RCKse^{st} = K(1 + RCs)e^{st} \\ \Rightarrow\; s &= -1/RC \\ \Rightarrow\; v(t) &= Ke^{-t/RC} \end{aligned}$$

Applying initial condition:

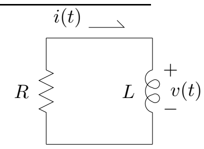
$$V_0 = v(0) = Ke^0 = K$$

$$\Rightarrow \boxed{v(t) = V_0\, e^{-t/RC}, t \geq 0}$$

Graphically:



RL response to initial conditions:



$i(0) = I_0$

$i(t)\big|_{t\geq 0} =$???

KCL yields (for $t \geq 0$)

$$\begin{aligned} 0 &= i(t) + v(t)G \\ &= i(t) + GL\frac{di(t)}{dt} \quad \text{. . . a differential equation} \end{aligned}$$

Trying $i(t) = Ke^{st}, t \geq 0$

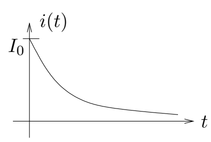
$$\begin{aligned} 0 &= Ke^{st} + GLKse^{st} = K(1+GLs)e^{st} \\ \Rightarrow\ s &= -1/GL \\ \Rightarrow\ i(t) &= Ke^{-t/GL} \end{aligned}$$

Applying initial condition:

$$I_0 = i(0) = Ke^0 = K$$

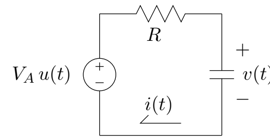
$$\Rightarrow \boxed{i(t) = I_0\, e^{-t/GL}, t \geq 0}$$

Graphically:



Dual of the RC case!

RC response to step function & initial condition:



$v(0) = V_0$

$v(t)\big|_{t\geq 0} =$???

Main idea: $v(t) = \underbrace{v_N(t)}_{\text{natural}} + \underbrace{v_F(t)}_{\text{forced}}$ (superposition!)

Natural response:

diff eq: $0 = RC\frac{dv_N(t)}{dt} + v_N(t), \quad t \geq 0$

postulate $v_N(t) = K_1 e^{st}, \quad t \geq 0$

$\Rightarrow v_N(t) = K_1 e^{-t/RC}, \quad t \geq 0$

Forced response:

diff eq: $V_A = RC\frac{dv_F(t)}{dt} + v_F(t), \quad t \geq 0$

postulate $v_F(t) = K_2, \quad t \geq 0$

$\Rightarrow v_F(t) = V_A, \quad t \geq 0$

Together

$$v(t) = V_A + K_1 e^{-t/RC}, \quad t \geq 0$$

Applying initial condition:

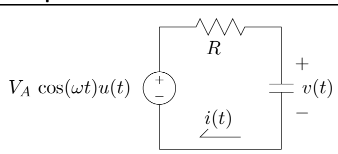
$$v(0) = V_0 \quad \Rightarrow \quad K_1 = V_0 - V_A$$

Thus

$$\boxed{v(t) = V_A + (V_0 - V_A)e^{-t/RC}, \quad t \geq 0}$$

What about an RL circuit?

RC response to causal sinusoid with initial condition:



$v(0) = V_0$

$v(t)\big|_{t\geq 0} =$???

Again, we use $v(t) = \underbrace{v_N(t)}_{\text{natural}} + \underbrace{v_F(t)}_{\text{forced}}$

Natural response:

diff eq: $0 = RC\frac{dv_N(t)}{dt} + v_N(t), \quad t \geq 0$

$\Rightarrow v_N(t) = K_1 e^{-t/RC}, \quad t \geq 0$

Forced response:

diff eq: $V_A \cos(\omega t) = RC\frac{dv_F(t)}{dt} + v_F(t), \quad t \geq 0$

postulate $v_F(t) = K_2 \cos(\omega t + \phi) \quad , \quad t \geq 0$
$\qquad = a\cos(\omega t) + b\sin(\omega t)$

solving diff eq gives $a = \frac{V_A}{1+(\omega RC)^2}, \quad b = \frac{\omega RC V_A}{1+(\omega RC)^2}$

Together

$$v(t) = K_1 e^{-t/RC} + a\cos(\omega t) + b\sin(\omega t), \quad t \geq 0$$

Applying initial condition:

$$v(0) = V_0 \quad \Rightarrow \quad K_1 = V_0 - a$$

Thus, for $t \geq 0$,

$$\boxed{v(t) = \Big(V_0 - \frac{V_A}{1+(\omega RC)^2}\Big)e^{-t/RC} + \frac{V_A}{1+(\omega RC)^2}\big(\cos(\omega t) + \omega RC\sin(\omega t)\big)}$$

Sinusoidal steady-state response:

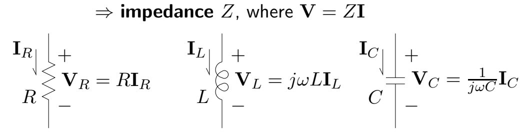
In an RLC circuit sinusoidally driven at frequency ω, all current and voltage waveforms are sinusoidal with frequency ω and thus completely described by their $magnitudes\ and\ phases.$

$\Rightarrow$ **phasors**

$$V_0\cos(\omega t + \phi) = \text{Re}\{V_0 e^{j(\omega t+\phi)}\} = \text{Re}\{\underbrace{V_0 e^{j\phi}}_{\text{phasor}\mathbf{V}} \cdot e^{j\omega t}\}$$

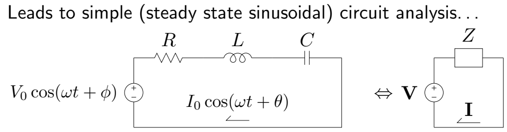
Passive circuit elements can be described in terms of their effect on current and voltage phasors:

$\Rightarrow$ **impedance** Z, where $\mathbf{V} = Z\mathbf{I}$



$Z_R = R \qquad Z_L = j\omega L \qquad Z_C = \frac{1}{j\omega C}$

Leads to simple (steady state sinusoidal) circuit analysis. . .



$$\begin{aligned} Z &= R + (j\omega C)^{-1} + j\omega L \\ \mathbf{V} &= V_0 e^{j\phi} \\ \Rightarrow \mathbf{I} &= I_0 e^{j\theta} = \mathbf{V}/Z \end{aligned}$$

. . . much easier than diff eq method!