Design Strategies for First-Order Cascades:

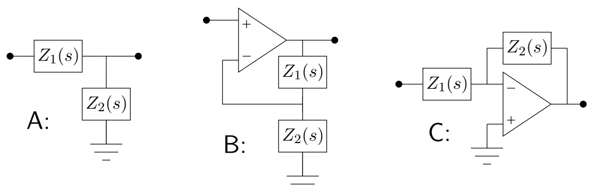
- Say desired $T(s)$ can be broken into first-order terms:

$$T(s) = T_1(s)T_2(s)\cdots T_N(s)$$

$$\text{where} \quad T_i(s) = \pm\frac{K_i s + b_i}{s + a_i}$$

$$\text{and} \quad K_i \geq 0, \quad a_i \geq 0, \quad b_i \geq 0.$$

- To design $T_i(s)$, one chooses a circuit configuration from
  - A. voltage divider: $T_i(s) = \frac{Z_2(s)}{Z_1(s)+Z_2(s)}$
  - B. non-inverting op-amp: $T_i(s) = \frac{Z_1(s)+Z_2(s)}{Z_2(s)}$
  - C. inverting op-amp: $T_i(s) = -\frac{Z_2(s)}{Z_1(s)}$

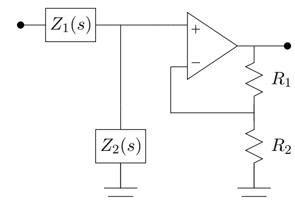


  and a sub-circuit configuration from
  1. series RL $Z_k(s) = L_k s + R_k$
  2. series RC $Z_k(s) = \frac{1}{C_k s} + R_k$
  3. parallel RC $Z_k(s) = (C_k s + \frac{1}{R_k})^{-1}$
  4. parallel RL. $Z_k(s) = (\frac{1}{L_k s} + \frac{1}{R_k})^{-1}$

- Element values can then be scaled. Remember that R and L values scale together, while R and C values scale inversely.

Phil Schniter ECE-301 Design Strategies

- When choosing the circuit configuration, keep in mind that
  - A. voltage divider:
    - unfriendly input/output loading
    - passive: requires no op-amps
    - restrictions: $K_i \leq 1, b_i \leq a_i$
    - can cascade with gain stage to mitigate loading problems and element restrictions:



$$T_i(s) = G_i \frac{\frac{K_i}{G_i}s + \frac{b_i}{G_i}}{s + a_i}$$

$$G_i = \frac{R_1 + R_2}{R_2}$$

  - B. non-inverting op-amp:
    - friendly input/output loading
    - active: requires an op-amp
    - restrictions: $K_i \geq 1, b_i \geq a_i$
  - C. inverting op-amp:
    - unfriendly input loading, friendly output loading
    - active: requires an op-amp
    - no restrictions on K_i, a_i, b_i
    - introduces negative gain

A.1: Voltage Divider Using Series-RL:

$$
\begin{aligned}
T_i(s) &= \frac{Z_2(s)}{Z_1(s)+Z_2(s)} \text{ where } \begin{cases} Z_1(s) = L_1 s + R_1 \\ Z_2(s) = L_2 s + R_2 \end{cases} \\
&= \frac{L_2 s + R_2}{(L_1+L_2)s + (R_1+R_2)} \\
&= \frac{Ks+b}{s+a} \\
\Rightarrow L_2 &= K \\
\Rightarrow R_2 &= b \\
\Rightarrow L_1 &= 1-K \\
\Rightarrow R_1 &= a-b
\end{aligned}
$$

restrictions : $K \leq 1, \quad b \leq a$

note : $K = 0 \;\Rightarrow\; L_2$ disappears

$K = 1 \;\Rightarrow\; L_1$ disappears

$b = 0 \;\Rightarrow\; R_2$ disappears

$b = a \;\Rightarrow\; R_1$ disappears

A.2: Voltage Divider Using Series-RC:

$$
\begin{aligned}
T_i(s) &= \frac{Z_2(s)}{Z_1(s)+Z_2(s)} \text{ where } \begin{cases} Z_1(s) = R_1 + \frac{1}{C_1 s} \\ Z_2(s) = R_2 + \frac{1}{C_2 s} \end{cases} \\
&= \frac{R_2 + \frac{1}{C_2 s}}{(R_1+R_2) + (\frac{1}{C_1 s} + \frac{1}{C_2 s})} \\
&= \frac{R_2 s + \frac{1}{C_2}}{(R_1+R_2)s + (\frac{1}{C_1} + \frac{1}{C_2})} \\
&= \frac{Ks+b}{s+a} \\
\Rightarrow R_2 &= K \\
\Rightarrow C_2 &= \frac{1}{b} \\
\Rightarrow R_1 &= 1-K \\
\Rightarrow C_1 &= \frac{1}{a-b}
\end{aligned}
$$

restrictions : $K \leq 1, \quad b \leq a$

note : $K = 0 \;\Rightarrow\; R_2$ disappears

$K = 1 \;\Rightarrow\; R_1$ disappears

$b = 0 \;\Rightarrow\; C_2$ disappears

$b = a \;\Rightarrow\; C_1$ disappears

A.3: Voltage Divider Using Parallel-RC:

$$
\begin{aligned}
T_i(s) &= \frac{Z_2(s)}{Z_1(s)+Z_2(s)} \text{ where } \begin{cases} Z_1(s) = (C_1 s + \frac{1}{R_1})^{-1} \\ Z_2(s) = (C_2 s + \frac{1}{R_2})^{-1} \end{cases} \\
&= \frac{(C_2 s + \frac{1}{R_2})^{-1}}{(C_1 s + \frac{1}{R_1})^{-1} + (C_2 s + \frac{1}{R_2})^{-1}} \\
&= \frac{(C_1 s + \frac{1}{R_1})}{(C_2 s + \frac{1}{R_2}) + (C_1 s + \frac{1}{R_1})} \\
&= \frac{C_1 s + \frac{1}{R_1}}{(C_2 + C_1)s + (\frac{1}{R_2} + \frac{1}{R_1})} \\
&= \frac{Ks+b}{s+a} \\
\Rightarrow C_1 &= K \\
\Rightarrow R_1 &= \frac{1}{b} \\
\Rightarrow C_2 &= 1-K \\
\Rightarrow R_2 &= \frac{1}{a-b}
\end{aligned}
$$

restrictions : $K \le 1, \quad b \le a$

note : $K=0 \;\Rightarrow\; C_1$ disappears

$K=1 \;\Rightarrow\; C_2$ disappears

$b=0 \;\Rightarrow\; R_1$ disappears

$b=a \;\Rightarrow\; R_2$ disappears

A.4: Voltage Divider Using Parallel-RL:

$$
\begin{aligned}
T_i(s) &= \frac{Z_2(s)}{Z_1(s)+Z_2(s)} \text{ where } \begin{cases} Z_1(s) = (\frac{1}{L_1 s} + \frac{1}{R_1})^{-1} \\ Z_2(s) = (\frac{1}{L_2 s} + \frac{1}{R_2})^{-1} \end{cases} \\
&= \frac{(\frac{1}{L_2 s} + \frac{1}{R_2})^{-1}}{(\frac{1}{L_1 s} + \frac{1}{R_1})^{-1} + (\frac{1}{L_2 s} + \frac{1}{R_2})^{-1}} \\
&= \frac{(\frac{1}{L_1 s} + \frac{1}{R_1})}{(\frac{1}{L_2 s} + \frac{1}{R_2}) + (\frac{1}{L_1 s} + \frac{1}{R_1})} \\
&= \frac{\frac{1}{L_1} + \frac{1}{R_1}s}{(\frac{1}{L_1} + \frac{1}{L_2}) + (\frac{1}{R_1} + \frac{1}{R_2})s} \\
&= \frac{Ks+b}{s+a} \\
\Rightarrow R_1 &= \frac{1}{K} \\
\Rightarrow L_1 &= \frac{1}{b} \\
\Rightarrow R_2 &= \frac{1}{1-K} \\
\Rightarrow L_2 &= \frac{1}{a-b}
\end{aligned}
$$

restrictions : $K \le 1, \quad b \le a$

note : $K=0 \;\Rightarrow\; R_1$ disappears

$K=1 \;\Rightarrow\; R_2$ disappears

$b=0 \;\Rightarrow\; L_1$ disappears

$b=a \;\Rightarrow\; L_2$ disappears

B.1: Non-Inverting Op-Amp Using Series-RL:

$$T_i(s) = \frac{Z_1(s)+Z_2(s)}{Z_2(s)} \text{ where } \begin{cases} Z_1(s) = L_1 s + R_1 \\ Z_2(s) = L_2 s + R_2 \end{cases}$$

$$= \frac{(L_1+L_2)s + (R_1+R_2)}{L_2 s + R_2}$$

$$= \frac{Ks+b}{s+a}$$

$$\Rightarrow L_2 = 1$$

$$\Rightarrow R_2 = a$$

$$\Rightarrow L_1 = K-1$$

$$\Rightarrow R_1 = b-a$$

restrictions : $K \geq 1, \quad b \geq a$

note $K=1 \;\Rightarrow\; L_1$ disappears

$a=0 \;\Rightarrow\; R_2$ disappears

$b=a \;\Rightarrow\; R_1$ disappears

B.2: Non-Inverting Op-Amp Using Series-RC:

$$T_i(s) = \frac{Z_1(s)+Z_2(s)}{Z_2(s)} \text{ where } \begin{cases} Z_1(s) = R_1 + \frac{1}{C_1 s} \\ Z_2(s) = R_2 + \frac{1}{C_2 s} \end{cases}$$

$$= \frac{(R_1+R_2) + (\frac{1}{C_1 s} + \frac{1}{C_2 s})}{R_2 + \frac{1}{C_2 s}}$$

$$= \frac{(R_1+R_2)s + (\frac{1}{C_1} + \frac{1}{C_2})}{R_2 s + \frac{1}{C_2}}$$

$$= \frac{Ks+b}{s+a}$$

$$\Rightarrow R_2 = 1$$

$$\Rightarrow C_2 = \frac{1}{a}$$

$$\Rightarrow R_1 = K-1$$

$$\Rightarrow C_1 = \frac{1}{b-a}$$

restrictions : $K \geq 1, \quad b \geq a$

note : $K=1 \;\Rightarrow\; R_1$ disappears

$a=0 \;\Rightarrow\; C_2$ disappears

$b=a \;\Rightarrow\; C_1$ disappears

B.3: Non-Inverting Op-Amp Using Parallel-RC:

$$\begin{aligned}
T_i(s) &= \frac{Z_1(s)+Z_2(s)}{Z_2(s)} \text{ where } \begin{cases} Z_1(s) = (C_1 s + \frac{1}{R_1})^{-1} \\ Z_2(s) = (C_2 s + \frac{1}{R_2})^{-1} \end{cases} \\
&= \frac{(C_1 s + \frac{1}{R_1})^{-1} + (C_2 s + \frac{1}{R_2})^{-1}}{(C_2 s + \frac{1}{R_2})^{-1}} \\
&= \frac{(C_2 s + \frac{1}{R_2}) + (C_1 s + \frac{1}{R_1})}{(C_1 s + \frac{1}{R_1})} \\
&= \frac{(C_2 + C_1)s + (\frac{1}{R_2} + \frac{1}{R_1})}{C_1 s + \frac{1}{R_1}} \\
&= \frac{Ks+b}{s+a} \\
\Rightarrow C_1 &= 1 \\
\Rightarrow R_1 &= \frac{1}{a} \\
\Rightarrow C_2 &= K-1 \\
\Rightarrow R_2 &= \frac{1}{b-a}
\end{aligned}$$

restrictions : $K \geq 1, \quad b \geq a$

note : $K = 1 \;\Rightarrow\; C_2$ disappears

$a = 0 \;\Rightarrow\; R_1$ disappears

$b = a \;\Rightarrow\; R_2$ disappears

B.4: Non-Inverting Op-Amp Using Parallel-RL:

$$\begin{aligned}
T_i(s) &= \frac{Z_1(s)+Z_2(s)}{Z_2(s)} \text{ where } \begin{cases} Z_1(s) = (\frac{1}{L_1 s} + \frac{1}{R_1})^{-1} \\ Z_2(s) = (\frac{1}{L_2 s} + \frac{1}{R_2})^{-1} \end{cases} \\
&= \frac{(\frac{1}{L_1 s} + \frac{1}{R_1})^{-1} + (\frac{1}{L_2 s} + \frac{1}{R_2})^{-1}}{(\frac{1}{L_2 s} + \frac{1}{R_2})^{-1}} \\
&= \frac{(\frac{1}{L_2 s} + \frac{1}{R_2}) + (\frac{1}{L_1 s} + \frac{1}{R_1})}{(\frac{1}{L_1 s} + \frac{1}{R_1})} \\
&= \frac{(\frac{1}{L_1} + \frac{1}{L_2}) + (\frac{1}{R_1} + \frac{1}{R_2})s}{\frac{1}{L_1} + \frac{1}{R_1}s} \\
&= \frac{Ks+b}{s+a} \\
\Rightarrow R_1 &= 1 \\
\Rightarrow L_1 &= \frac{1}{a} \\
\Rightarrow R_2 &= \frac{1}{K-1} \\
\Rightarrow L_2 &= \frac{1}{b-a}
\end{aligned}$$

restrictions : $K \geq 1, \quad b \geq a$

note : $K = 1 \;\Rightarrow\; R_2$ disappears

$a = 0 \;\Rightarrow\; L_1$ disappears

$b = a \;\Rightarrow\; L_2$ disappears

C.1: Inverting Op-Amp Using Series-RL:

$$T_i(s) = -\frac{Z_2(s)}{Z_1(s)} \text{ where } \begin{cases} Z_1(s) = L_1 s + R_1 \\ Z_2(s) = L_2 s + R_2 \end{cases}$$
$$= -\frac{L_2 s + R_2}{L_1 s + R_1}$$
$$= -\frac{Ks + b}{s + a}$$

$$\Rightarrow L_2 = K$$
$$\Rightarrow R_2 = b$$
$$\Rightarrow L_1 = 1$$
$$\Rightarrow R_1 = a$$

restrictions : none

note : $K = 0 \Rightarrow L_2$ disappears
$a = 0 \Rightarrow R_1$ disappears
$b = 0 \Rightarrow R_2$ disappears

C.2: Inverting Op-Amp Using Series-RC:

$$T_i(s) = -\frac{Z_2(s)}{Z_1(s)} \text{ where } \begin{cases} Z_1(s) = R_1 + \frac{1}{C_1 s} \\ Z_2(s) = R_2 + \frac{1}{C_2 s} \end{cases}$$
$$= -\frac{R_2 + \frac{1}{C_2 s}}{R_1 + \frac{1}{C_1 s}}$$
$$= -\frac{R_2 s + \frac{1}{C_2}}{R_1 s + \frac{1}{C_1}}$$
$$= -\frac{Ks + b}{s + a}$$

$$\Rightarrow R_2 = K$$
$$\Rightarrow C_2 = \frac{1}{b}$$
$$\Rightarrow R_1 = 1$$
$$\Rightarrow C_1 = \frac{1}{a}$$

restrictions : none

note : $K = 0 \Rightarrow R_2$ disappears
$a = 0 \Rightarrow C_1$ disappears
$b = 0 \Rightarrow C_2$ disappears

C.3: Inverting Op-Amp Using Parallel-RC:

$$
\begin{aligned}
T_i(s) &= -\frac{Z_2(s)}{Z_1(s)} \text{ where } \begin{cases} Z_1(s) = (C_1 s + \frac{1}{R_1})^{-1} \\ Z_2(s) = (C_2 s + \frac{1}{R_2})^{-1} \end{cases} \\
&= -\frac{(C_2 s + \frac{1}{R_2})^{-1}}{(C_1 s + \frac{1}{R_1})^{-1}} \\
&= -\frac{C_1 s + \frac{1}{R_1}}{C_2 s + \frac{1}{R_2}} \\
&= -\frac{Ks + b}{s + a}
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow C_1 &= K \\
\Rightarrow R_1 &= \frac{1}{b} \\
\Rightarrow C_2 &= 1 \\
\Rightarrow R_2 &= \frac{1}{a}
\end{aligned}
$$

restrictions : none

note : $K = 0 \;\Rightarrow\; C_1$ disappears

$a = 0 \;\Rightarrow\; R_2$ disappears

$b = 0 \;\Rightarrow\; R_1$ disappears

C.4: Inverting Op-Amp Using Parallel-RL:

$$
\begin{aligned}
T_i(s) &= -\frac{Z_2(s)}{Z_1(s)} \text{ where } \begin{cases} Z_1(s) = (\frac{1}{L_1 s} + \frac{1}{R_1})^{-1} \\ Z_2(s) = (\frac{1}{L_2 s} + \frac{1}{R_2})^{-1} \end{cases} \\
&= -\frac{(\frac{1}{L_2 s} + \frac{1}{R_2})^{-1}}{(\frac{1}{L_1 s} + \frac{1}{R_1})^{-1}} \\
&= -\frac{\frac{1}{L_1 s} + \frac{1}{R_1}}{\frac{1}{L_2 s} + \frac{1}{R_2}} \\
&= -\frac{\frac{1}{L_1} + \frac{1}{R_1} s}{\frac{1}{L_2} + \frac{1}{R_2} s} \\
&= -\frac{Ks + b}{s + a}
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow R_1 &= \frac{1}{K} \\
\Rightarrow L_1 &= \frac{1}{b} \\
\Rightarrow R_2 &= 1 \\
\Rightarrow L_2 &= \frac{1}{a}
\end{aligned}
$$

restrictions : none

note : $K = 0 \;\Rightarrow\; R_1$ disappears

$a = 0 \;\Rightarrow\; L_2$ disappears

$b = 0 \;\Rightarrow\; L_1$ disappears